Just Intonation in Ancient Indian Theory: Its Applications to Modal and Harmonic Music[1]

I. FOREWORD

1. Abstract

The ancient musical theory of India, comprehensively presented in the earliest Sanskrit treatises on music[2], and which concerns itself with the notion of consonance, with musical scales and their systematic arrangements preserving the enharmonic differences between tonal positions of notes, constitutes the most incisive formulation of the just intonation system that has ever been proposed. It is also capable of providing a considerable quantity of information useful in the practice of modal music, as well as in that of harmonic music.

[2] *Natyashastra* (IVth c. before J.-C.), *Dattilam* (uncertain date) and *Brihaddeshi* (VIIth c. after J.-C.).

This article proposes a mathematical model of the *grama-murchana* system (fundamental scales and their modal transformations) which, once cut out and assembled, constitutes a circular calculating disc of considerable interest for the study of musical scales. This model was conceived for the modal music of India, but it can be easily adapted to harmonic music.

This ancient theory of *musical geometry* is the ideal starting point for any research on the phenomenon of intonation in the classical music of North India. Not only does it propose a logical enharmonic arrangement of the scales of ragas, in accordance with the requirements of melodic movements, but one can also show that this theory provides the key to a coherent explanation (founded on simple psycho-acoustic phenomena) of the traditional practice consisting of interpreting each raga at a particular hour.

Moreover, the *grama-murchana* system permits us to answer certain questions relating to just intonation in the context of Western classical harmonic music, for it establishes the criteria for choosing the enharmonic positions of notes as a function of chord progressions and modulations.

2. Historical perspective

Although the Sanskrit treatises written later continued, for the most part, to describe the ancient system, this seems to have ceased to be clearly interpreted, probably as early as the XIIth century at the time of the *Sangitaratnakara* of Sarangadeva[3]. The cultural upheavals provoked by the Muslim invasions certainly contributed to accelerating this process, but it is curious to note, however, that the line of research which succeeded in supplanting the ancient system originates from South India (*Swaramelakalanidhi* of Ramamatya, 1550 after J.-C.)[4].

This new theory, based on the idea of classifying ragas with the help of several (10 to 72) scale types (*mela-s*) includes, it is true, and for completely acceptable historical reasons, a large number of musical scales which are not described by the ancient *grama-murchana* system (whose basis of the classification system of melodic entities is constituted solely of two fundamental scales (*grama-s*)). But it renounces taking into consideration the ideal of internal equilibrium of mutual consonances between the two tetrachords of these musical scales, and consequently it gives no indication on the enharmonic positions of their notes.

Following the discovery of the existence of the ancient system by Sir William Jones[5], numerous researchers have attempted to reconstruct and interpret this theory in the absence of a living tradition which would provide authority[6]. To cast some light on this problem, two centuries of research and theoretical speculation have been necessary, in which the most eminent Indian and European specialists have participated. The reconstitution of the *grama-murchana* system is the fruit of the encounter of two cultures, given that all interest in questions of musical scales in just intonation was suspended in Europe as in India — but for different reasons. It is only in recent decades, however, that this reconstitution has been able to be achieved[7].

3. Hypotheses of the present study

The study that follows shows that if one considers that the set of 23 tonal positions of a complete cycle[8] of fourths and fifths constructed from a single base note (Do) or Sa[9] corresponds to the system of 22 *shruti-s*[10] which divides the octave according to the ancient system, and if one establishes the fundamental scales Sa-*grama* and Ma-*grama* from these tonal positions following the instructions of the texts, the transformations of these fundamental scales into different modes (*murchana-s*) (obtained by shifting the tonic to the different notes of the *grama-s*) then result in an acoustically ideal set of enharmonic positions of the notes of diatonic scales, chromatic modes, and enharmonic chords to which is associated the chromatic scale.

The principal quality of these scales defined by the *grama-murchana* system resides in the equilibrium of their mutual consonances between the two tetrachords, equilibrium highlighted by the play of melodic movements. We shall see that the major and minor scales of Zarlino form part of this set. It is interesting to note that M. Klarenz Barlow has recently shown that computer analysis of the "harmonicity" of the enharmonic chords of diatonic scales results in a preferential choice of tonal positions that correspond to those of the *grama-murchana* system thus interpreted[11].

4. Limits of our study

This study is limited to the framework of "musical geometry" without claiming to provide definitive answers on the adequate use of musical intervals. It may be that tonal positions which seem theoretically valid are not accepted as such by the ear, for example in the presence of a harmonic-rich tonic generator. We have recently carried out experiments with some expert Indian musicians using a *tanpura* and the "shruti harmonium", a programmable polyphonic electronic organ of great precision realised by M. Bernard Bel; these experiments have shown that the ear seems to prefer tonal positions diminished by 4 or 5 *schismas*[12] for the major third and sixth in Pythagorean positions, and by 2 or 3 for the positions of most minor notes, whilst the sensible notes seem to undergo more or less the attraction of their resolution note.

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[3] O. Thakur, *Pranava-bharati* (Banaras: chez l'Auteur, 1956), p. 64-65. [4] P. L. Sharma, "The ancient *grama* system and its distortion in the medieval times", *Journal of the Music Academy*, XXXII (1961), 37-40. [5] Sir W. Jones, *On the Musical Modes of the Hindoos* (1784), reproduced in Raja S. M. Tagore, *Hindu Music from Various Authors* (Calcutta: Chowkhamba Sanskrit Series Office, Varanasi, 1875). [6] R. S. Bhandarkar, "Contributions to the study of ancient Hindu music", *The Indian Antiquary*, 1912; E. J. Arnold, "A perspective on the problem of raga *shrutis* and *grama-murchana* analysis", *Journal of the Sangeet Natak Academy* (New Delhi: Sangeet Natak Academy, January 1981). [7] F. Framji, *The Theory and Practice of Indian Music* (Poona, 1938). [8] We replace certain complex Pythagorean ratios with simple harmonic ratios. For example, the ratio 8192/6561 is replaced by 5/4, harmonic major third. [9] As far as modal music is concerned, the names of notes are understood in relative value. [10] We shall examine further the reason for the numbers 22 and 23. [11] K. Barlow, "Bus journey to Paramstron", *Feedback Papers*, 21-23 (Cologne: Feedback Studio, December 1980), 115. [12] 1 schisma = 0.84 savart.

II. A MATHEMATICAL MODEL OF THE SHRUTI-SWARA-GRAMA-MURCHANA SYSTEM

1. Introduction

This section is devoted to the description of our mathematical model of the musical acoustic system *shruti-swara-grama-murchana* proposed by Bharata (*Natyashastra*, Ch. 28, 21 - 36), Dattila (*Dattilam* I, 6A - 30A), Sarangadeva (*Sangitaratnakara*, 3:8 - 5:11), etc. The basic idea consists of mounting a mobile disc graduated according to the tonal positions of the fundamental scales (*grama-s*) on a circular logarithmic graph representing the division of the octave into *shruti-s*. It should be noted that the Sanskrit term "*shruti*" is used indifferently to designate sometimes an enharmonic *position*, sometimes an *interval*[13]. In the first case, *shruti* designates the tonal position of a note in the octave in relation to the tonic (starting note, Sa or Do). In the second case, the term *shruti* designates an interval which, depending on the case, can take three distinct values (see II.2).

To construct this calculating disc, one will cut out the mobile disc printed on page 33 and paste it on cardboard; one will paste likewise on a rigid support the graph represented on page 35. One will assemble these two parts with the help of a fastener or a tubular rivet allowing the rotation of the mobile part.

The model permits immediate observation of the enharmonic relations which characterise the structure of musical scales in the just intonation system. The error due to the *schisma* is sometimes visible, but should be ignored. By positioning any *swara* (note) of the mobile disc (disc of the *grama*) on the Sa (Do) of the *shruti* graph, one immediately obtains the list of enharmonic positions of the notes (*shruti-s*) which compose the modal scale (*murchana*) constructed from the note chosen on the mobile disc.

In fact, the principal scale types employed by Hindustani classical music form a principal series which can be deduced directly from the harmonic structure of the fundamental scales (*grama-s*). This property is summarised in the table of the "extended series of *murchana-s*" (modes) relative to the scales which derive from the fundamental scale of Fa (Ma-*grama*) and that of Do (Sa-*grama*). By "fundamental scale" we understand a particular structure of notes, a notion which must be distinguished from that of "scale type" (*murchana*, *mela*, *thaat*) or "mode". The mode is simply the application of the structure obtained by transposition of the tonic (Sa). The two fundamental scales cited are two forms from which flows the dorian mode; they differ only by a microtonal interval for the position of the note of the fifth degree of the *grama* (Pa³ or Pa⁴).

2. Description of the fixed part of the model (page 35)

The circular crown represents the octave divided according to the system of *shruti-s*, determined, to the nearest *schisma*, by the cycles of ascending and descending fifths. The interval symbols are as follows:

— C represents the syntonic comma (81/80);
— M represents the chromatic semitone (25/24);
— L represents the limma (256/243).

These intervals indicate the tonal distance between the points of manifestation (*sphota*) of the notes. These positions are designated by abbreviations of the Sanskrit names of notes (Sa, Ri, Ga etc...) followed by exponents (1), (2), (3), (4). Exponents (1) and (2) indicate the two enharmonic positions of "minor" notes (*komal*), and (3) and (4) those of "major" notes (*tivra*). To symbolise concisely the procedure to follow for tuning (or defining mathematically) the positions of the enharmonic division of the octave, we have borrowed from Jacques Dudon symbols thus defined[15]:

1.	△ₙ	représente la N-ième quinte ascendante à partir de Sa.
2.	ᴡ	représente la N-ième quinte descendante à partir de Sa.
3.	⌒	représente la tierce majeure (harmonique) ascendante à partir de Sa.
4.	⌣	représente la tierce majeure (harmonique) descendante à partir de Sa.
5.	⩕ ou �	représente la N-ième quinte ascendante, ou descendante à partir de la tierce majeure au-dessus de Sa.
6.	△ ou ᴡ	représente la N-ième quinte ascendante, ou descendante à partir de la tierce majeure au-dessous de Sa.

3. Validity of this division from the point of view of ancient treatises

No ancient treatise provides in fact mathematical indication on the value of one *shruti*. These treatises are content to mention the number of *shruti-s* which separate two *swara-s* (notes) in the fundamental scales (*grama-s*), as well as the fifth ratios between the notes of the *grama-s* where appropriate. The traditional validity of the proposed division rests on three arguments:

1. The acoustic value of the intervals of fifth and fourth (13 and 9 *shruti-s* respectively) in the harmonic structure of scales is used as a criterion of consonance by the Sanskrit treatises. The positions that we have established on the graph belong to a complete cycle of fourths and fifths (to the nearest *schisma*); these positions divide the octave into twenty-two *shruti-s*, as we shall see subsequently. Any other tonal position which could be proposed would imply the use of new, more complex axioms, or would introduce again intervals which have no simple numerical significance. The positions used in musical practice may, certainly, differ from these theoretical positions, which, in fact, are only references for the intellect and not necessarily for the ear.

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[13] "Enharmonic position-note-fundamental scale-modal transposition". [14] M. Lath, *A Study of Dattilam* (New Delhi: Impex India, 1978), p. 203-206. [15] E. J. Arnold, "Notes on the construction of a simple *shruti-swara-grama-yantra*", *American Institute of Indian Studies Quarterly Newsletter* (New Delhi: A.I.I.S., 1974), p. 28-30.

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2. If one determines from our model the *shruti-s* of the fundamental scales (Ma and Sa *grama-s*) respecting the values of the intervals (in *shruti-s*) indicated by the texts[16], no modal transposition (*murchana*) causes a tonal position to intervene which is exterior to the original graph. Consequently, this division of the octave into twenty-two *shruti-s*, and the system of fundamental scales Ma-*grama* and Sa-*grama* which flows from it, form a coherent and valid whole as much from the acoustic point of view as from the geometric.

3. It remains finally to show that the twenty-three intervals formed by the graph constitute in fact a system of "twenty-two *shruti-s*". The experience of the "two *vina-s*" (lutes) mentioned by Bharata[17] permits us to appreciate auditively the intervals of 4 *shruti-s*, 3 *shruti-s*, and 2 *shruti-s*, and to establish the respective equivalences of these intervals with the major tone (9/8), the minor tone (10/9), and the diatonic semitone (16/15) known in the West. Similarly, from the two *grama-s*, it permits us to appreciate auditively the values of a comma (in the first lowering of one *shruti*), of a limma (in the second lowering of one *shruti*), of a chromatic semitone (in the third lowering of one *shruti*), and of a comma once more (in the fourth lowering of one *shruti*)[18]. The "*shruti*" itself is therefore not an interval of fixed dimension[19].

It should be noted that the limma (256/243) equals a chromatic semitone (25/24) plus a comma (81/80). This is where the ambiguity between the numbers twenty-two and twenty-three lies: in effect, between Ma¹ (Fa) and Pa⁴ (Sol) of the fixed graph of *shruti-s*, interval of a major tone (4 *shruti-s* as can be seen on the mobile disc of the *grama-s*), the graph shows five enharmonic positions (C + M + C + M + C), where one should count four *shruti-s* (C + M + C + L, or indeed: L + C + M + C).

4. Description of the mobile disc (disc of grama, page 33)

The periphery of the *grama* disc is divided into twelve notes which constitute a particular chromatic scale. This scale is established according to the following principles: given twenty-two enharmonic positions to tune the eleven notes of the chromatic scale — excluding the base (Do), or Sa, which remains fixed — there exist 2^{11}, that is 2,048 different ways, to tune the chromatic scale. Most of them, however, contain several dissonant fifths (wolf fifths, 40/27). Twelve of them are optimal from the point of view of consonance (having only one wolf fifth). The chromatic scale chosen on the periphery of the disc is the only one among the twelve which can contain all the enharmonic positions of the fundamental scale of (Fa), Ma-*grama*, to which the Sanskrit treatises make reference. This is why we shall call it "the Ma-*grama* chromatic scale". This scale is defined by a complete tour of the cycle of fourths (neglecting the *schisma* problem).

The fundamental scale of (Do), Sa-*grama*, differs from that of Ma-*grama* only by the position of the fifth degree of the *grama* (Pa⁴ instead of Pa³). This position (Pa³) is indicated on the periphery, as well as the positions *cyuta* Sa (Do — 1c.) and *cyuta* Ma (Fa — 1c.) which do not form part of the Ma-*grama* chromatic scale but which are used in the procedure called "transposition of a comma" (*sadharana*) (see III.3).

The second circle towards the interior indicates the notes of the diatonic fundamental scales Ma-*grama* and Sa-*grama*. The unaltered notes (*shuddha swara-s*) are marked by solid rays, and the altered notes (*vikrita swara-s*) by dots. These notes function principally as variants of the first in certain melodic phrases where the altered note is used in the ascending movement, and the natural note in the descending movement[20].

We have indicated on the inner crown the numbers of *shruti-s* which justify the position of the notes of the *grama-s* according to the texts[21].

5. Internal harmony of the fundamental scales (grama-s)

The internal harmony, or acoustic structure, of the two fundamental scales (*grama-s*) is underlined in the smallest circle of the mobile disc. The fundamental scale of (Fa) and that of (Do) (Ma and Sa *grama-s*) are composed of two chains of consonances of fifths which are offset one from the other by a harmonic major third.

It should be noted that the fundamental scale of (Do), Sa-*grama*, possesses this characteristic property that its fifth degree (Pa⁴) belongs to the same chain of consonances as (Do) Sa, whilst in the fundamental scale of (Fa), Ma-*grama*, the fifth degree (Pa³) belongs to the other chain. The fifth degree, which, in any case, must figure in every fundamental scale, must be chosen amongst these two possibilities. From the theoretical point of view, these two choices are equally valid, and this note therefore constitutes the "critical point" of the internal harmony of the two scales.

If the fifth degree of the *grama* belongs to the chain of (Do) Sa, (case of Sa-*grama*) then the notes (Do), (Ré), (Mi♭), (Fa), or Sa, Re³, Ga¹, Ma¹, are in mutual consonance with (Sol), (La), (Si♭), (Do₂) or Pa⁴, Dha³, Ni¹, Sa. This scale is formed of "disjoint" tetrachords, between (Fa) and (Sol) or Ma¹ and Pa⁴ and "conjoint" to (Do), Sa.

If, on the other hand, (Sol), Pa³ belongs to the chain of (Ré), Re³, (case of Ma-*grama*) then the notes (Ré), (Mi♭), (Fa), (Sol), or Re³, Ga¹, Ma¹, Pa³, are in mutual consonance with (La), (Si♭), (Do₂), (Ré₂), or Dha³, Ni¹, Sa, Re³. This scale is formed of disjoint tetrachords between (Sol) and (La) or Pa³ and Dha³, and conjoint to (Ré), Re³. Evidently, the difference between these two cases resides in the displacement of the harmonic equilibrium, which implies different options for melodic movement.

We touch here on an extremely important phenomenon as regards the relations between the harmonic skeleton and the melodic movement of ragas. In general, one can affirm that two melodically parallel phrases (and of similar rhythmic morphologies) imply consonance relations obeying the same parallelism. However, in the case where these parallelisms are not formal, the system of relations between melodic movement and harmonic structure is more subtle, and only an experimental study of musical practice will permit us to deepen these questions theoretically. This new approach is at the heart of our current research[22].

However, the genius of the ancient system of analysis of musical scales resides in the fact that the crucial choice of the note situated at the critical position (Sol) or Pa of the fundamental scales (*grama-s*) is found by the play of modal transposition (*murchana*) at critical points of all the diatonic scales (and their chromatic extensions) linked by the same sequence, and which form the basis of the great families of ragas.

6. The application of the grama-murchana system to contemporary Hindustani music

The principal scale types which are commonly recognised in the classical music of North India belong to the logic of the *grama-murchana* system. The diatonic modes (*thaat* or *mela*) of current Hindustani music: *kalyan*, *bilaval*, *khamaj*, *kaphi*, *asavari*, and *bhairavi* (equivalent respectively to the modes of (Fa), (Do), (Sol), (Ré), (La) and (Mi) on the white keys of the piano) are the scale types (*murchana-s*) which begin respectively with the notes of the fundamental scales: Ga¹, Ni¹, Ma¹, Sa, Pa³ and Re³. (We advise the reader to use the circular model to account for this mechanism.) The scale types which contain one or two intervals of augmented second, *marva*, *purvi*, *todi*, *lalit*, and even *bhairao* to a certain extent, belong to an extension of the first-order parameters of the *grama-murchana* system[23].

To link these scale types to the logic of the *grama*, it is necessary to relax somewhat the rules of the ancient system. In the first place, one must use the *murchana-s* beginning with the altered notes (*vikrit swara-s*: Ga³ and Ni³). Moreover, one must accept *murchana-s* beginning on a note exterior to the *grama-s*, but belonging to the scale that we have accepted as a logical extension of the base scale: the Ma-*grama* chromatic scale (see III.3). It is necessary, finally, to suppress some unaltered notes of the *grama-s* when they are replaced by the altered notes[24].

The table opposite, which summarises the principal sequence of scale types of the *grama-murchana* system extended to what precedes, shows that the principal modes (*thaat-s*) of the music of North India constitute a logical suite of scale types whose internal harmony flows from that of the *grama-s*.

For the *murchana-s* beginning on each note of the Ma-*grama* chromatic scale, the table indicates which mode (*thaat*) is produced:

La série étendue *GRAMA-MURCCHANA*
ET LES PRINCIPALES ÉCHELLES TYPES
DE LA MUSIQUE HINDUSTANI CONTEMPORAINE

Etape:	Note de départ: transposition modale de la gamme chromatique Ma-grama	(1) *Echelle fondamentale inaltérée	(2) Echelle altérée (antara Ga)	(3) Echelle altérée (antara Ga kakali Ni)	Note(s) à ajouter à la gamme**	Note(s) à retrancher à la gamme**
1.	Sa (Do)	kaphi	khamaj	bilaval	--	--
2.	Ma^1 (Fa)	khamaj	bilaval	kalyan	--	--
3.	Ni^1 (Si^b)	bilaval	kalyan	marva	3: N^1	3: S
4.	Ga^1 (Mi^b)	kalyan	marva	purvi	2: G^1 3: G^1 N^1	2: M^1 3: M^1 S
5.	Dha^1 (La^b)	marva	purvi	todi	1: D^1 2: D^1 G^1 3: D^1 G^1	1: N^1 2: N^1 M^1 3: S M^1
6.	Re^1 (Ré^b)	purvi	todi	--	1: R^1 D^1 2: R^1 D^1	1: G^1 N^1 2: G^1 M^1 N^1
7.	Ma^3 (Fa)	todi	--	a. lalit b. bhairav	1: M^3 R^1 3a: M^3 3b: M^3 R^1	1: N^1 G^1 3a: D^3 G^1 3b: D^3 S
8.	Ni^3 (Si)	--	--	a. lalit b. bhairav c. purvi d. bhairavi	3a: G^1 N^1 3b: G^1 M^3 3c: G^1 M^3 3d: M^3	3a: R^3 D^3 3b: R^3 M^1 D^3 3c: R^3 D^3 3d: M^1
9.	Ga^3 (Mi)	--	--	a. todi b. bhairavi	3a: -- 3b: --	3a: D^3 R^3 3b: --
10.	Dha^3 (La)	--	bhairavi	asavari	--	--
11.	Re^3 (Ré)	bhairavi	asavari	kaphi	--	--
12.	Pa^3 (Sol)	asavari	kaphi	khamaj	--	--

* Le terme « échelle fondamentale » désigne indifféremment l'échelle Sa-*grama* ou l'échelle Ma-*grama*.
** Les nombres renvoient aux colonnes (1), (2) et (3).

1. from the unaltered fundamental scale;

2. from the fundamental scale altered with *antara gandhara* (Ga³);

3. from the fundamental scale altered with *antara grandhara* (Ga³) and *kakali nishad* (Ni³).

The table finally indicates the notes which are added and those which, where appropriate, are suppressed from the fundamental scale.

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[16] According to the treatises, there are 3-4-2-4-3-4-2-4 *shruti-s* between the notes Sa-Re-Ga-Ma-Pa-Dha-Ni of the Ma-*grama* and 3-4-2-4-3-2-4 *shruti-s* between the same notes of the Sa-*grama* (see *Dattilam*, 1.11A-15B). [17] Bharata, *Natyasastra*, commented by Abhinavagupta, edited by M. Ramakrishna Kavi, IV, *Gaekwad's Oriental Series*, n° 145 (Baroda, 1964), ch. 28:26. [18] A. A. Bake, "Bharata's experiment with two vinas", *Bulletin of the School of Oriental and African Studies*, XX (London: The School of Oriental and African Studies, 1957), p. 61-67. [19] Cf. K. B. Deval, *Music of East and West Compared* (Poona, 1908). [20] Cf. Bharat, *op. cit.*, 21:35-36. [21] Cf. note 16. [22] E. J. Arnold et G. B. Bel, *Acoustic study of modal structures and melodic movements* (Bourges: chez les Auteurs, 1980), p. 9 et 21. [23] These scales are called "chromatic modes" by the Greek tradition. [24] It should be noted that the rule of *avahashi madhyama* (the invariable use of the Ma note of the *grama-s*) is occasionally transgressed (see *Dattilam*, 20A-B).

THE EXTENDED GRAMA-MURCHANA SERIES AND THE PRINCIPAL SCALE TYPES OF CONTEMPORARY HINDUSTANI MUSIC

[THIS IS TABLE: A complex table showing modal transpositions with columns for departure note, fundamental scale, altered scale, notes to add/remove, etc. The table includes 12 rows numbered 1-12 with various musical notations and scale types like Sa (Do), Ma¹ (Fa), Ni¹ (Si♭), etc.]

- The term "fundamental scale" designates indifferently the Sa-grama scale or the Ma-grama scale.
  ** The numbers refer to columns (1), (2) and (3).

First, one must use the *murchana-s* beginning with the altered notes (*vikrit swara-s*: Ga³ and Ni³). Moreover, one must accept *murchana-s* beginning on a note exterior to the *grama-s*, but belonging to the scale that we have accepted as a logical extension of the base scale: the Ma-*grama* chromatic scale (see III.3). It is necessary, finally, to suppress some unaltered notes of the *grama-s* when they are replaced by the altered notes.

The table shows that the principal modes (*thaat-s*) of North Indian music constitute a logical sequence of scale types whose internal harmony derives from that of the *grama-s*.

III. THE TIME OF INTERPRETATION OF RAGAS AND THE GRAMA-MURCHANA SYSTEM

1. Introduction

The idea that one can associate various melodic types with the hours of the day which are suitable for their interpretation, an idea resting on the sensation that certain musical modes evoke certain feelings or a specific ambiance compatible with certain hours of the day or night, is certainly one of the most striking facts of the classical music of North India[25]. It is curious that such an insolent property of this musical system has never been studied in depth[26]. Although this association is very ancient — mentioned for the first time by Matanga in the VIIIth century — the essential of what constitutes today the theory of the association of ragas with their hours of interpretation consists of some fragmentary and barely sketched rules[27].

As Deva has underlined, most of these rules imply considerations of scale, of position of the *vadi swara* (the tension note of the raga) or even of melodic movement of the raga[28]. If one limits oneself to questions of scale types, the problem can be approached at two levels: the *chromatic* perspective and the *transilient* perspective[29].

2. The chromatic perspective

Starting from the Ma-*grama* chromatic scale, one finds by modal transposition the twelve chromatic scales of optimal consonance. It is logical to consider as the beginning of this series the scale whose twelve notes occupy the lowest potential microtonal positions (*shruti-s*); in this case the scale which is found at the end of the series is that whose twelve notes occupy the highest positions. The note Do (Sa) evidently always occupies the same position. The first scale is composed of the cycle of fourths (Sa *murchana* of the Ma-*grama* chromatic scale); the last is composed of the cycle of fifths (Pa³ *murchana* of the Ma-*grama* chromatic scale). This series of ordered transitions (from "minor" to "major") of the *shruti-s* of the chromatic scale is indicated in the table opposite, placed in parallel with its aesthetic significance.

[THIS IS TABLE: Progressive Majorisation of Shrutis showing 12 steps with chromatic scale formations and aesthetic values from "plus sombre" (darkest) to "plus claire" (brightest)]

It should be noted that, up to the seventh step inclusive, the transition only affects the "major" notes (*tivra*), and that beyond it affects only the "minor" notes (*komal*).

The chromatic scale of the seventh step is the most active of this series: it has all its minor notes (*komal*) in extreme minor position (*atikomal*), and simultaneously all its major notes (*tivra*) in extreme major position (*tivratar*). All the *shruti-s* of this particular scale belong to the Pythagorean family. The Pythagorean microtonal positions are the extreme limits of the clear manifestations of each degree of the octave and their frequency ratios are generally more complex than those of harmonic positions[30]. These complex ratios imply more "internal tensions", and therefore, on the aesthetic plane, greater severity.

MAJORISATION PROGRESSIVE DES *SHRUTI*-S DE LA GAMME CHROMATIQUE

étape	murcchana de la Ma-grama chromatique à partir de		Do / S	Ré♭ / R̲	Ré / R	Mi♭ / G̲	Mi / G	Fa / M̲	Fa♯ / M	Sol / P	La♭ / D̲	La / D	Si♭ / N̲	Si / N	valeur esthétique
1.	Sa	(Do)	°	−	−	−	−	−	−	−	−	−	−	−	plus sombre
2.	Ma^1	(Fa)	°	−	−	−	−	−	−	+	−	−	−	−	.
3.	Ni^1	(Si♭)	°	−	+	−	−	−	−	+	−	−	−	−	.
4.	Ga^1	(Mi♭)	°	−	+	−	−	−	−	+	−	+	−	−	.
5.	Dha^1	(La♭)	°	−	+	−	+	−	−	+	−	+	−	−	.
6.	Re^1	(Ré♭)	°	−	+	−	+	−	−	+	−	+	−	+	région de tension maximale
7.	Ma^3	(Fa♯)	°	−	+	−	+	−	+	+	−	+	−	+	
8.	Ni^3	(Si)	°	+	+	−	+	−	+	+	−	+	−	+	.
9.	Ga^3	(Mi)	°	+	+	−	+	−	+	+	+	+	−	+	.
10.	Dha^3	(La)	°	+	+	+	+	−	+	+	+	+	−	+	.
11.	Re^3	(Ré)	°	+	+	+	+	−	+	+	+	+	+	+	.
12.	Pa^3	(Sol)	°	+	+	+	+	+	+	+	+	+	+	+	plus claire

−−gamme chromatique formée−−

Légende :
— = position potentielle la plus basse.
+ = position potentielle la plus haute.
° = position fixe.

It is curious to note that the minor and major notes are in a state of extreme tension within the same chromatic scale. The "cold" notes (the minors) are at their coldest whilst the "warm" notes (the majors) are at their warmest. This chromatic scale is therefore a transitional scale which possesses an important degree of instability, of "stress". This type of transition, implied on the aesthetic plane by the tensions within the chromatic scales, can be put in parallel with the period of transition of diurnal and nocturnal activity: the rising and setting of the sun.

"Similarly, the *sandhiprakasha* of the evening (dusk) represents the warmest moment of the night, from which the night becomes cooler, just as the *sandhiprakasha* of the morning (dawn) represents the coldest moment of the day, from which it becomes warmer. By analogy, the sixth and seventh steps contain both the major notes that are "warmest", which "cool down" gradually if one goes back towards the first step, as well as the minor notes that are "coldest", which "warm up" progressively if one goes in the direction of the twelfth step."

"Thus we must expect that the ragas of the night use scales which, as the night advances, are born from chromatic scales evolving towards the first step. Similarly we must expect that the ragas of the day use scales which, as the day progresses, are born from chromatic scales evolving towards the twelfth step"[31]. This relationship is expressed graphically by the diagram on page 25.

3. The point of view of transilient scales

To envisage the question of the time from the point of view of transilient scales of seven to nine notes, it is necessary to refer to the table of scales of the extended *grama-murchana* series (see page 20). This table indicates the natural affinity between the notes of the Ma-*grama* chromatic scale and certain particular scale types whose structure is linked to that of the fundamental scales (Ma-*grama* and Sa-*grama*).

We must give here some technical details that the reader can neglect if he prefers to refer directly to the diagram which summarises the relationship between hours and scale types linked to *grama-s* (see page 27).

The extended *grama-murchana* series of scales given on page 20 is recurrent: from the second step (Pa[3] *murchana*), one can imagine a thirteenth which would be the beginning of a new cycle. However, from the fact that we begin a new cycle of descending fifths, the thirteenth *murchana* would be constructed from the Sa of the *grama* lowered by a comma, which is called *cyuta* Sa. The reader will be able to refer to the circular model to note that this scale is none other than that of step 1 transposed towards the top by a comma. We note that the series "progresses", not like a closed cycle, but according to a spiral. But to approach the problem of the time of interpretation in relation with the principal sequence of scale types (the extended *grama-murchana* series), it is necessary to have the freedom to begin the sequence at step 7 and to pass directly from step 12 to step 1, and vice-versa as we shall see subsequently. To close the cycle, it therefore suffices to proceed to what the texts call *sadja-sadharana*[32], an operation which consists of raising by a comma the *cyuta* Sa of the fundamental scale, which amounts to using the original Sa.

step – étape	murcchna	
1	Sa	
2	Ma	
3	Ni	
4	Ga	
5	Dha	
6	Re	
7	Pa = M'a	
8	Ni	
9	Ga	
10	Dha	
11	Re	
12	Pa	
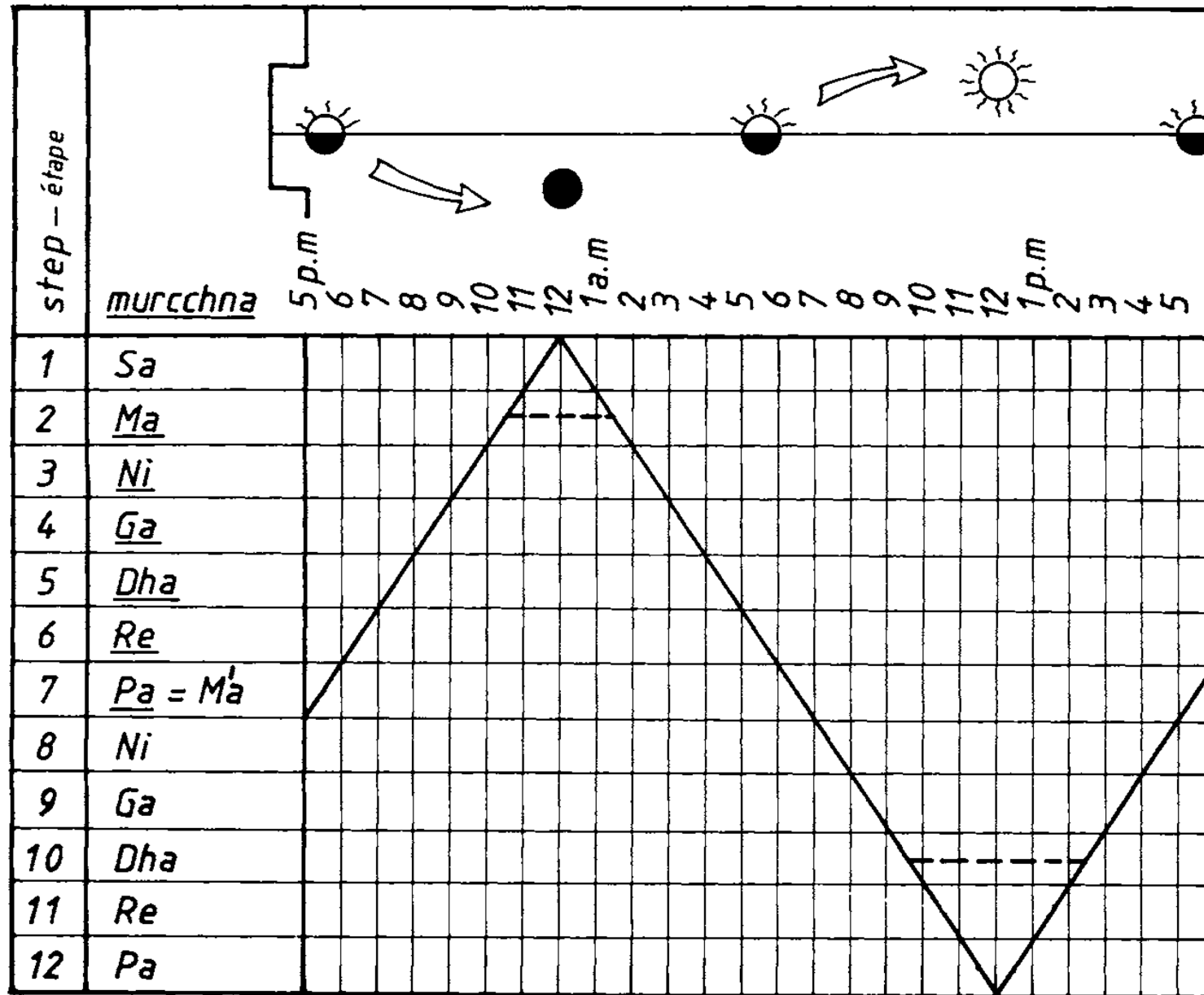
A noter que les échelles correspondant aux étapes 1 et 12 utilisent respectivement Pa 3 et Ma 2, qui forment des quintes et quartes imparfaites avec Sa ; c'est pourquoi elles ne peuvent pas être adoptées pratiquement en présence d'un générateur de tonique accordé sur Sa (Do). D'où le tracé en pointillé.

By adopting the same principle as in the chromatic point of view, namely that the seventh step (which also corresponds to the most active scale type) must represent the *sandhiprakasha* — sunset and sunrise[33], during the night one must pass progressively from step 7 to step 1, and during the day from step 7 to step 12. However, to follow the natural evolution through the different scale types, after midnight when the night advances, and after midday when the day progresses, one must be able to pass from step 12 to step 1 (and vice-versa) directly. From the chromatic point of view this possibility is to be rejected because of the discontinuity that it would introduce in the majorisation (or minorisation) progressive of the *shruti-s* (see table, p. 22). But from the point of view of transilient scales, the continuity between the modes: *khamaj*, *kaphi*, *asavari*, *bhairavi* which have 2, 3, 4 and 5 minor notes (*komal*) respectively, is evident. The passage from *kaphi* to *asavari*, perfectly logical as regards the evolution of scale types and of "cosmic ambiance" following the time of day (or night), must not however imply a discontinuity (passing from *murchana-s* of the fundamental scale from Sa, then Pa^3) as regards the microtonal possibilities, as we have already noted.

Also is it appropriate to distinguish two "driving impulses": one is the continuity of the evolution of chromatic scales which underlie the diatonic scale types, the other is that of the sequence of scale types linked to the (diatonic) fundamental scale (*grama*). It is indispensable that these two progressive evolutions be respected during the mentioned passages: after midnight and after midday. For this, it suffices to accept certain scale types belonging to the *grama-murchana* series, but for which the starting note of their *murchana* "definitive" is not the same according to whether one considers the basic chromatic scale to determine its enharmonic positions. Thus, for example, around 23 hours one should interpret the ragas of the *kaphi* mode (mode of Ré), and after midnight those of the *asavari* mode (mode of La). But the first, which are deduced by modal transposition from Sa, have all their notes on the lowest potential positions, whilst the last, formed from Pa^3 *murchana*, would have all their notes on the highest positions. Such an arrangement for *asavari* might suit the day, but certainly not in full night!

The solution therefore consists of following the series of modal transpositions of the fundamental diatonic scale Ma-*grama* or Sa-*grama* to determine the scale types, and those of the Ma-*grama* chromatic scale as regards the microtonal positions of the notes (*swara-s*) of these scale types. This is what is summarised by the diagram on page 27.

4. Times of interpretation, murchana-s, and ragas

In accordance with these indications, the table on pages 28-29 summarises, for 85 ragas approximately, the times of interpretation, the modal transpositions (*murchana-s*) corresponding, as well as the agreement of their scale types following the system of 22 *shruti-s*. The notes provided with two exponents, one between parentheses, are those which result from the Sa-*grama*/Ma-*grama* choice, and which therefore constitute the "critical point" of the scale (see page 19).
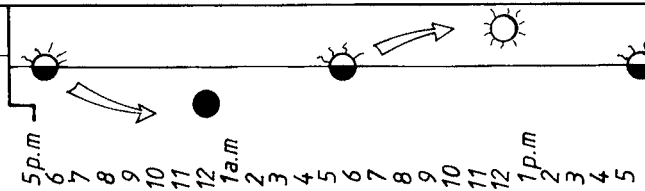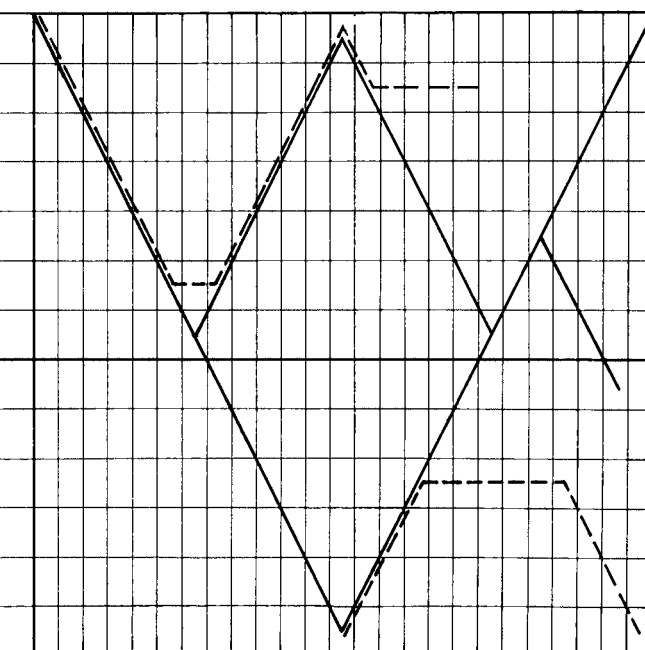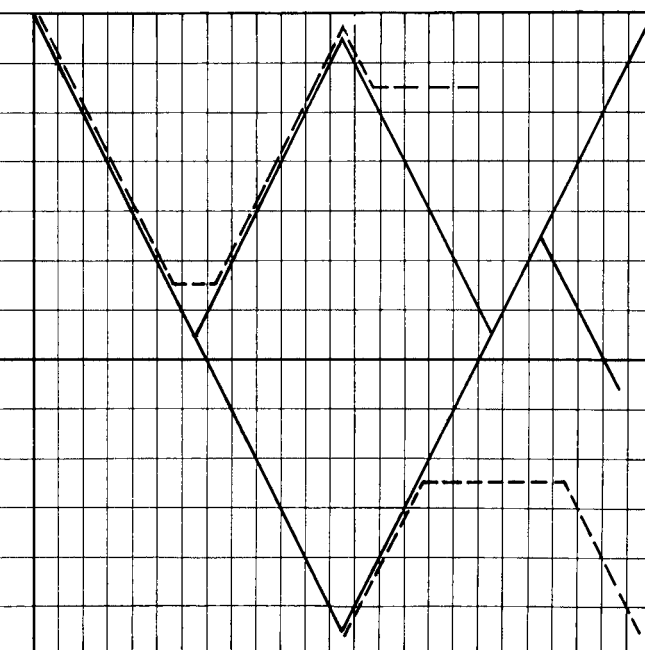
These indications should be considered, not as a definitive system, but rather as a sort of guide: the reader is invited to experiment himself auditively with the implications of the *grama-murchana* system. The indications of interpretation time should not claim an abusive rigidity: for musicians, this question is rather a matter of intuition as of tradition, and its rules are more flexible than the table might suggest.

However, what is quite remarkable is that the indications of interpretation times that we have systematically deduced from the ancient *grama-murchana* theory are almost entirely in conformity with those that tradition has conveyed to our days. This is the indubitable proof of the extreme consistency and pertinence of the *grama-murchana* system, as of the continuity of the raga system through the ages!

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[25] See the fourth paragraph of this section. [26] B. C. Deva, "Ragas and their time", *Psychoacoustics of Music and Speech* (Madras: The Music Academy, 1967), 140. [27] K. N. Das, "Time theory of the ragas (North Indian)", *Journal of the Music Academy*, XVII (Madras: The Music Academy, 1951), 69-82. [28] Deva, art. cit., 140. [29] We shall use this term to designate a musical scale in which certain notes have been omitted. [30] Arnold, "Notes...", art. cit., 30. [31] *Ibid.*, p. 26-27. [32] R. K. Shringy, P. L. Sharma, *Sangita Ratnakara of Sarngadeva: text and translation* (Benares: Motilal Banarsidass, 1978), vol. 1, 5:7-9. [33] In fact the process begins an hour before dawn or dusk.

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step – étape	Ma-grama chromatic murcchna starting from: à partir de:	"Main sequence" scales – Séquence principale			time axis (5 p.m – 5)
		shuddh grama	vikrit grama (antara)	vikrit grama (antara-kakali)	
7	Má	Todi	——	(Purvi) (Marva)	
6	Re	Purvi	Todi	——	
5	Dha	Marva	Purvi	Todi	
4	Ga	Kalyan	Marva	Purvi	
3	Ni	Bilaval	Kalyan	Marva	
2	Ma	Khamaj	Bilaval	Kalyan	
1	Sa	Kaphi	Khamaj	Bilaval	
12	Pa	Asavari	Kaphi	Khamaj	
11	Re	Bhairavi	Asavari	Kaphi	
10	Dha	Komal – pancam (*)	Bhairavi	Asavari	
9	Ga	——		Todi Bhairavi	
8	Ni	——	——	Lalit Bhairav	
7	Ma	Todi	——	Lalit Bhairav	

En trait continu, la série d'échelles transilientes ; en pointillé, celle des échelles chromatiques.
(*) Voir tableau page 28.

heure	étape	échelle du *grāma* à partir de	échelles-type correspondantes	Ma-*grāma* mūrcchanā chromatique à partir de	ragas
soir 5 - 6	7.	Ma³	S R1*(2) G1 G4 M1 M4 P4 D1 D4 N4 (todi/purvi/marva)	Ma³	Purvi Triveni
6 - 7	6.	Re¹	S R1 G1 G4 M3(4) P4 D1 D4 N1 N4 (purvi/todi/marva)	Re¹	Shri Puriyadhanashri Maligaura
7 - 8	5.	Dha¹	S R1 G1 G4 M3 P4 D1 D4 N3(4) (marva/purvi/todi)	Dha¹	Jetashri Marva Jait
8 - 9	4.	Ga¹	S R1 R4 G3(4) M3 P4 D1 D4 N3 (kalyan/marva/purvi)	Ga¹	Puriya Kalyan Shyam-kalyan Puriya-kalyan
9 - 10	3.	Ni¹	S R1 R4 G3 M1 M3 P4 D3(4) N3 (bilaval/kalyan/marva)	Ni¹	Bhupali Kamod Chayanat Hamir Durga
10-11	2.	Ma¹	S R3(4) G3 M1 M3 P4 D3 N1 N3 (khamaj/bilaval/kalyan)	Ma¹	Kedar Tilang Tilak-kamod Desh Jhinjhoti Khamaj Rageshri
11-12	1.	Sa	S R3(4) G1 G3 M1 P4 D3 N1 N3 (kaphi/khamaj/bilaval)	Ma¹	Jaijaivanti Kaphi Malgunji Bageshri Nayaki Kanhada
matin 12 - 1	12.	Pa³	S R3(4) G1 G3 M1 P4 D1 D3 N1 (asavari/kaphi/khamaj)	Ma¹	Darbari Kanhada Adana Kanhada Barva Malkauns
(ou)	2.	Sa	S R3(4) G3 M1 M3 P4 D3 N1 N3 (khamaj/bilaval/kalyan)	Ma¹	Bihag Samkara
1 - 2	11.	Re³	S R1 R4 G1 M1 P4 D1 D3(4) N1 (bhairavi/asavari/kaphi)	Ni¹	Malkauns Bairagi
(ou)	3.	Ni¹	S R1 R4 G3 M1 M3 P4 D3(4) N1 (bilaval/kalyan/marva)		Bhattiyar Sohini Hindol
(matin) 2 - 3	10.	Dha³	S R1 R4 G1 M1 M3 P4 D1 N1 (--/bhairavi/asavari)	Ga¹	Bairagi Sampurn-malkauns
(ou)	4.	Ga¹	S R1 R4 G3(4) M3 P4 D1 D4 N3 (kalyan/marva/purvi)	Ga¹	Paraj Sohini Vasant
3 - 4	9.	Ga³	S R1 G1 M1 M3 P4 D1 N1 N3(4) (--/bhairavi/todi)	Dha¹	Kalingada
(ou)	5.	Dha¹	S R1 G1 G4 M3 P4 D1 D4 N3(4) (marva/purvi/todi)	Dha¹	Lalit Paraj
4 - 5	8.	**Ni³**	S R1 G1 G4 M1 M3(4) P4 D1 N1 N4 (--/lalit/bhairao)	Re¹	Prabhat Lalit-pancam
(ou)	6.	Re¹	S R1 G1 M3(4) P4 D1 D4 N1 N4 (purvi/todi/marva)	Re¹	Jogiya
5 - 6	7.	Ma³	S R1(2) G1 G4 M1 M4 P4 D1 D4 N4 (todi/lalit/bhairao)	Ma³	Bhairao Shivmat-bhairao Vibhas (bhairao) Todi

heure	étape*	échelle du *grama* à partir de	échelles-type correspondantes	Ma-*grama* murcchana chromatique à partir de	ragas
6 - 7	8.	Ni^3	S R2 G1 G4 M1 M4 P4 D1(2) N1 N4 (todi/lalit/bhairao)	Ni^3	Vasant-mukhari Ramkali Ahir-bhairao
(ou)	6.	Re^1	S R1 G1 G3 M3(4) P4 D1 D4 N1 N4 (purvi/todi/--)	Re^1	Vibhas (purvi) Todi
7 - 8	9.	Ga^3	S R2 G1(2) M1 M4 P4 D2 N1 N4 (--/--/todi/bhairavi)	Ga^3	Gunkali Vilaskhani todi
(ou)	5.	Dha^1	S R1 G4 M3(4) P4 D1 D4 N4 (marva/purvi/todi)	Re^1	Ananda-bhairao Bhattiyar Vibhas (marva)
8 - 9	10.	Dha^3	S R2 R4 G2 M1 M4 P4 D2 N1(2) (--/bhairavi/asavari)	Dha^3	Shuddha-bhairavi Bhupali-todi
(ou)	4.	Ga^1	S R1 R4 G4 M3(4) P4 D1 D4 N4 (kalyan/marva/purvi)	Re^1	Hindol Deshkar
9 - 10	11.	Re^3	S R2 R4 G2 M1(2) P4 D2 D4 N2 (bhairavi/asavari/kaphi)	Re^3	Komal-asavari Bhairavi Gandhara
(ou)	3.	Ni^1	S R1 R4 G4 M1 M3(4) P4 D4 N4 (bilaval/kalyan/marva)	Re^1	Bilaval
10-11	12.	Pa^3	S R4 G2 G4 M1 P4 D2 D4 N1(2) (asavari/kaphi/khamaj)	Dha^3	'savari Devgandhar
(ou)	2.	Ma^1	S R4 G3 M1 M3(4) P4 D4 N1 N4 (khamaj/bilaval/kalyan)	Re^1	Deshi Bilaval
11-12	1.	Sa	S R4 G2 G4 M1 P4 D4 N1(2) N4 (kaphi/khamaj/bilaval)	Dha^3	Deshi Suha Sugharai Bilaval Sarang
après midi 12 - 1	2.	Ma^1	S R4 G4 M1 M4 P4 D4 N1(2) N4 (khamaj/bilaval/kalyan)	Dha^3	Madhyamad-sarang Brindavani-sarang Shuddha-sarang
1 - 2	3.	Ni^1	S R2 R4 G4 M1 M4 P4 D4 N4 (bilaval/kalyan/marva)	Dha^3	Suddha-sarang Gaud-sarang
2 - 3	2.	Ma^1	S R4 G4 M1 M4 P4 D4 N1(2) N4 (khamaj/bilaval/kalyan)	Dha^3	Suddha-sarang Gaud-sarang
(ou)	4.	Ga^1	S R2 R4 G4 M4 P4 D2 D4 N4 (kalyan/marva/purvi)	Dha^3	Brindivani-sarang Din-ki-puriya
3 - 4	1.	Sa	S R4 G1(2) G4 M1 P4 D4 N2 N4 (kaphi/khamaj/bilaval)	Ga^3	Bhimpalasi Dhanashri
(ou)	5.	Dha^1	S R2 G1(2) G4 M4 P4 D2 D4 N4 (marva/purvi/todi)	Ga^3	Din-ki-puriya Madhuvanti Patmanjari Multani
4 - 5	1.	Sa	S R4 G1(2) G4 M1 P4 D4 N2 N4 (kapni/khamaj/bilaval)	Ga^3	Bhimpalasi Patadip
(ou)	6.	Re^1	S R2 G1 M4 P4 D1(2) N1 N4 (purvi/todi/--)	Ni^3	Madhuvanti Multani

* Les chiffres entre parenthèses indiquent les positions enharmoniques qui dérivent du cinquième degré du Sa-*grama* (voir III.4).

IV. APPLICATION OF THE GRAMA-MURCHANA SYSTEM TO WESTERN CLASSICAL HARMONIC MUSIC

This study, conducted in collaboration with M. Pierre-Yves Asselin during the summer of 1980, demonstrates that the ancient Indian system can provide the key to the problem of enharmonic positions of notes in chord progressions or tonal modulations.

1. The offset of tonics in major and minor modes

We have observed experimentally, with the aid of the *shruti harmonium*, that the major and minor scales:

<div align="center">

majeure

$$\text{Do} \; \frac{8}{9} \; \text{Ré} \; \frac{10}{9} \; \text{Mi} \; \frac{16}{15} \quad \text{Fa} \; \frac{256}{243} \; (\text{Fa} \; \sharp) \; \frac{16}{15} \; \text{Sol} \; \frac{10}{9} \; \text{La} \; \frac{9}{8} \; \text{Si} \; \frac{16}{15} \; \text{Do}_2$$

$$\under!\!\underline{\qquad\quad \frac{9}{8} \quad\qquad}$$

mineure

$$\text{Do} \; \frac{16}{15} \; (\text{Ré}\flat) \; \frac{256}{243} \; \text{Ré} \; \frac{16}{15} \; \text{Mi}\flat \; \frac{10}{9} \; \text{Fa} \; \frac{9}{8} \; \text{Sol} \; \frac{16}{15} \; \text{La}\flat \; \frac{9}{8} \; \text{Si}\flat \; \frac{10}{9} \; \text{Do}_2$$

$$\frac{9}{8}$$

</div>

which define the relative positions of notes, chords in just intonation, cannot be constructed on the same chromatic scale, and that the just intonation system for harmonic music (as well as for modal music) actually uses twelve "optimally consonant" chromatic scales (which contain only one imperfect fifth) (see II.4).

However, the juxtaposition of the major and minor scales above does not belong to a chromatic scale of this series. The Do (etc.) major scale, for example, is not constructed on the same tonic as that of Do (etc.) minor: the first, second, and fifth degrees (notes common to these two scales) must be, in the case of the minor scale, lowered by a comma. In relation to the just intonation system, the Do minor scale must be deduced by modal transposition (change of *murchana*) from the Mi♭ major scale, just as the La minor scale is a modal transposition of the Do major scale. Thus, to move from Do major to Do minor, one must modulate by four descending successive fifths (Do-Fa-Si♭-Mi♭), then take the relative minor scale.

The effect of this process on the enharmonic positions of notes can be demonstrated using the circular model. We shall proceed as follows:

1. **First stage**: place the Ma¹ of the mobile disc on the Sa (Do) of the fixed part. Examine which tonal positions of the fixed part are opposite the marks of the mobile part:

Sa-Re³-Ga³-Ma¹-Pa³-Dha³-Ni¹-Ṡa

(which is the Sa-*grama* using the replacement note Ga³ (*antara gandhara*) [34]. These tonal positions are those of the Zarlino scale (Do major):

Sa-Re♭-Ga³-Ma¹-Pa³-Dha³-Ni³-Ṡa

with their Indian names.

2. **Second stage**: turn the mobile disc by a descending fifth; the Ma¹ of the mobile disc comes opposite the Ma¹ (Fa) of the fixed part. Thus, the Zarlino scale, which starts from Ma¹ on the mobile disc, has been transposed to Fa by a descending fifth.

3. **Third stage**: turn the disc again by another descending fifth, which places the Ma¹ of the mobile disc on the Ni¹ (Si♭) of the fixed part.

4. **Fourth stage**: proceed in the same way; the Ma¹ of the mobile disc is then on the Ga¹ (Mi♭) of the fixed part.

5. **Fifth stage**: take the relative minor mode of Mi♭, which gives the Do minor scale.

Summary: We have thus modulated from Do major to Do minor by four descending successive fifths and performed a modal transposition to the relative minor scale. The tonal positions indicated on the fixed part, opposite the marks of the mobile disc, indicate the Do minor scale starting from Sa (Do) of the fixed part: it should be noted that the enharmonic positions of Do (Sa), Ré (Re³), and Sol (Pa³) have been lowered by a comma.

We have studied experimentally the progression and modulation from Do major to Do minor:

1. in just intonation, by lowering Do, Ré, and Sol by a comma;

2. in just intonation, without changing the tonic of these two scales;

3. using equal temperament.

---

[34] In fact, the *grama-murchana* system serves, to explore musical scales, six *fundamental scales* (*grama-s*): Sa-*grama*, Ma-*grama*, and their two altered versions — using *antara gandhara* or *antara gandhara* and *kakali nishad*. It is beyond our scope to explain in detail the acoustic and "grammatical" significance of these *grama-s*, which satisfy differently the requirements of modal music and harmonic music.

Auditory verification has shown that the effect of this chord progression or scale modulation is markedly reinforced in the first case, in accordance with the requirements of the *grama-murchana* system. This also applies to other progressions or modulations.

2. Major and minor scales in just intonation, and their relationships to the Ma-grama and Sa-grama fundamental scales

Placing the Ma¹ of the mobile part of the model once more on the Sa of the fixed part to obtain the Zarlino major scale. As we emphasised in II.5, the "critical" note of the fundamental scale (*grama*), the *pancam*, or fifth degree of the *grama* (which may be raised or lowered by a comma, thus determining whether it is Sa-*grama* or Ma-*grama*) is found at different positions by modal transposition (change of *murchana*), as well as by transposition of tonality (modulation).

FONDATIONS CHROMATIQUES DU SYSTÈME DES TONALITÉS.

--monde majeur-- (tonalité + positions enharmoniques des accords) — *--monde mineur--* (tonalité + positions enharmoniques des accords)

tonalité	positions enharmoniques des accords			tonalité	positions enharmoniques des accords			Dbb / B# / C	Db / C#	Ebb / C## / D	Eb / D#	Fb / D## / E	Gbb / E# / F	Gb / F#	Abb / F## / G	Ab / G#	Bbb / G## / A	Bb / A#	Cb / A## / B	murc-chana
Dbb	Dbb-	Fb-	Abb°	Bbb	Bbb-	Dbb-	Fb-	-	-	(°)	-	-	-	-	°	-	-	-	-	Ma[2]
Abb	Abb°	Cb-	Ebb°	Fb	Fb-	Abb°	Cb-	-	-	°	-	-	-	-	°	-	(°)	-	-	Ni[2]
Ebb	Ebb°	Gb-	Bbb°	Cb	Cb-	Ebb°	Gb-	-	-	°	-	(°)	-	-	°	-	°	-	-	Ga[2]
Bbb	Bbb°	Db-	Fb°	Gb	Gb-	Bbb°	Db-	-	-	°	-	°	-	-	°	-	°	-	(°)	Dha[2]
Fb	Fb°	Ab-	Cb°	Db	Db-	Fb°	Ab-	-	-	°	-	°	-	(°)	°	-	°	-	°	Re[2]
Cb	Cb°	Eb-	Gb°	Ab	Ab-	Cb°	Eb-	-	(°)	°	-	°	-	°	°	-	°	-	°	Ma[4]
Gb	Gb°	Bb-	Db°	Eb	Eb-	Gb°	Bb-	-	°	°	-	°	-	°	°	(°)	°	-	°	Ni[4]
Db	Db°	F-	Ab°	Bb	Bb-	Db°	F-	-	°	°	(°)	°	-	°	°	°	°	-	°	Ga[4]
Ab	Ab°	C-	Eb°	F	F-	Ab°	C-	-	°	°	°	°	-	°	°	°	°	(°)	°	Dha[4]
Eb	Eb°	G°	Bb°	C	C-	Eb°	G°	-	°	°	°	°	(°)	°	°	°	°	°	°	Re[4]
Bb	Bb°	D°	F°	G	G°	Bb°	D°	(°)	°	°	°	°	°	°	°	°	°	°	°	Pa[4]
F	F°	A°	C°	D	D°	F°	A°	°	°	°	°	°	°	°	(°+)	°	°	°	°	Sa
C	C°	E°	G+	A	A°	C°	E°	°	°	(°+)	°	°	°	°	+	°	°	°	°	Ma[1]
G	G+	B°	D+	E	E°	G+	B°	°	°	+	°	°	°	°	+	°	(°+)	°	°	Ni[1]
D	D+	F#°	A+	B	B°	D+	F#°	°	°	+	°	(°+)	°	°	+	°	+	°	°	Ga[1]
A	A+	C#°	E+	F#	F#°	A+	C#°	°	°	+	°	+	°	°	+	°	+	°	(°+)	Dha[1]
E	E+	G#°	B+	C#	C#°	E+	G#°	°	°	+	°	+	°	(°+)	+	°	+	°	+	Re[1]
B	B+	D#°	F##+	G#	G#°	B+	D#°	°	(°+)	+	°	+	°	+	+	°	+	°	+	Ma[3]
F#	F#+	A#°	C#+	D#	D#°	F#+	A#°	°	+	+	°	+	°	+	+	(°+)	+	°	+	Ni[3]
C#	C#+	E#°	G#+	A#	A#°	C#+	E#°	°	+	+	(°+)	+	°	+	+	+	+	°	+	Ga[3]
G#	G#+	B#°	D#+	E#	E#°	G#+	B#°	°	+	+	+	+	°	+	+	+	+	(°+)	+	Dha[3]
D#	D#+	F#+	A#+	B#	B#°	D#+	F##+	°	+	+	+	+	(°+)	+	+	+	+	+	+	Re[3]
A#	A#+	C##+	E#+	F##	F##+	A#+	C##+	(°+)	+	+	+	+	+	+	+	+	+	+	+	Pa[3]
E#	E#+	G##+	B#+	C##	C##+	E#+	G##+	+	+	+	+	+	+	+	(‡+)	+	+	+	+	Sa[-]
B#	B#+	D##+	F##+	G##	G##+	B#+	D##+	+	+	(‡+)	+	+	+	+	++	+	+	+	+	Ma[-]

DISQUE MOBILE DE *GRAMA.*

Découper ici

GRAPHE FIXE DES SHRUTIS.

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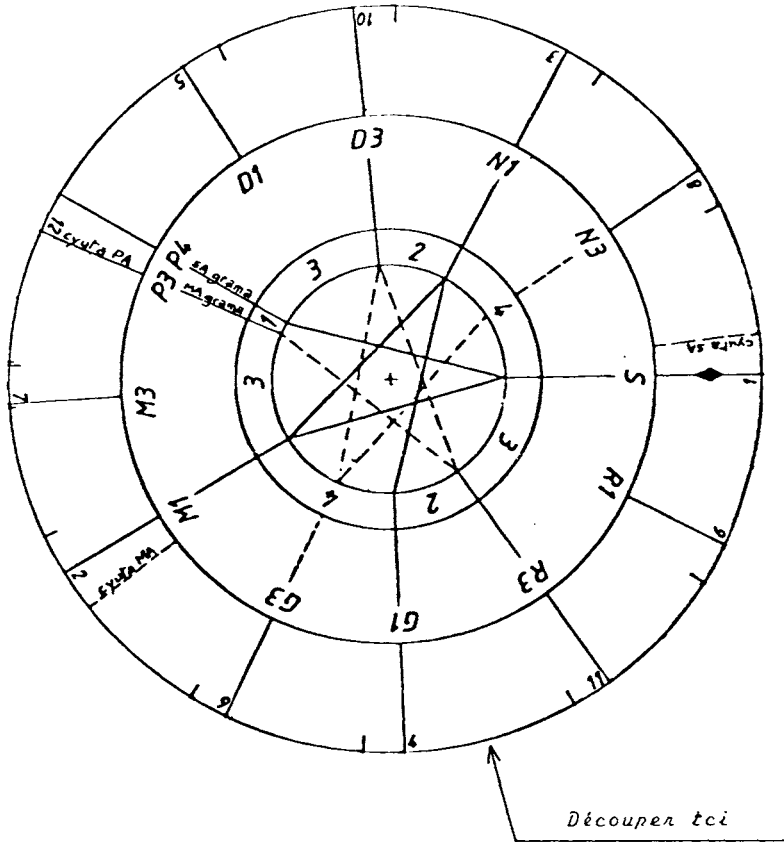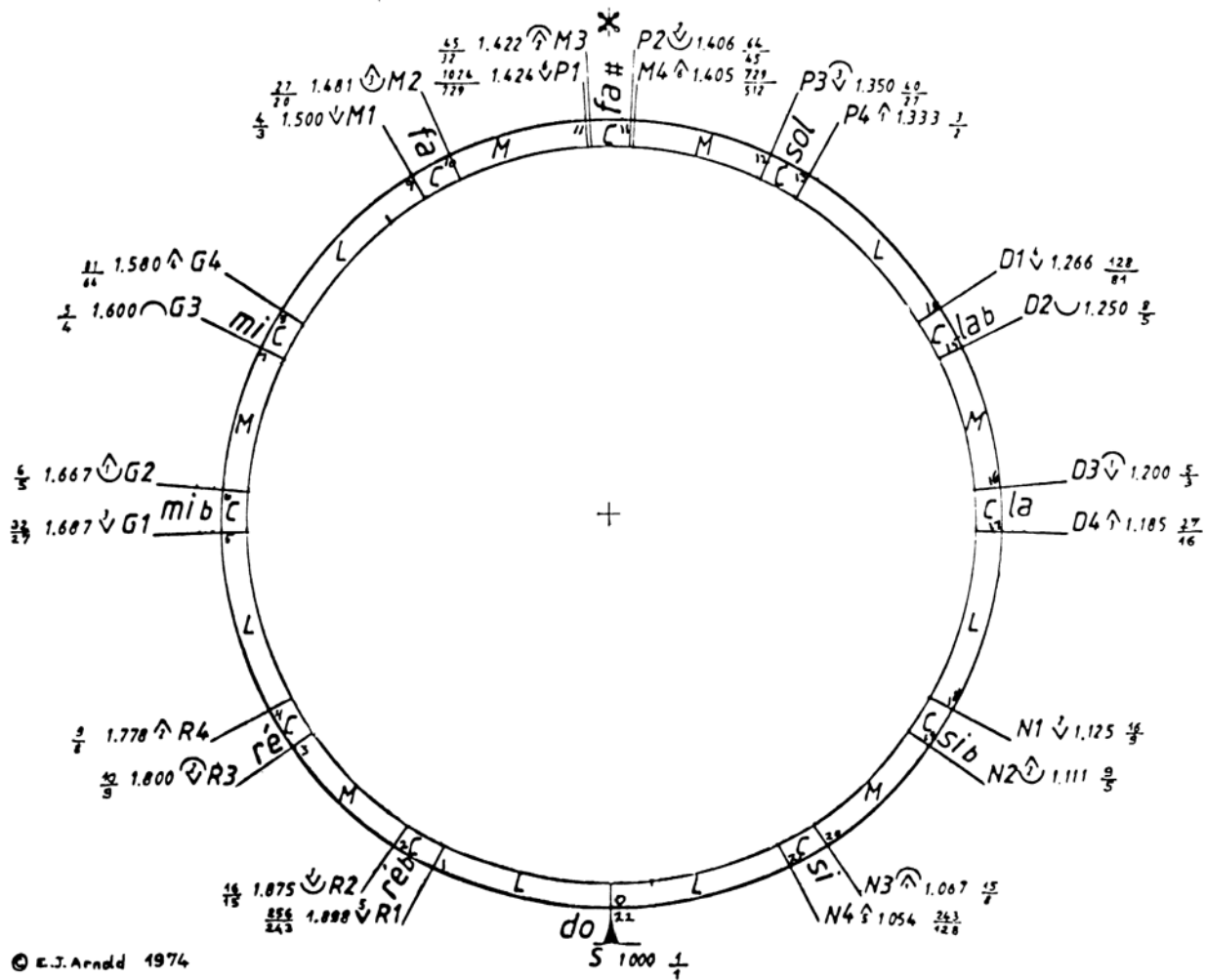
In the case of the Zarlino scale in harmonic music, the critical note should become (Ré) (Re³ or Re♭), and not (La), as one might commonly imagine. The note (La), should play the role of harmonic major third (5/4 ratio) in the (Fa) major chord, and should simultaneously be at the same enharmonic position to be a harmonic minor third (6/5) below (Do) for the relative minor scale (and its first-degree chord). The only case where the note (La) becomes critical is when the melodic relationship (Sol)-(La) should reflect that of (Do)-(Ré); that is to say when one is about to modulate towards (Sol) major, or when one is in the process of doing so. The note (Ré), by contrast, should be high (9/8) when it belongs to the (Sol) major chord, and low (10/9) when it belongs to the (Ré) minor chord. In this latter case, it is in the correct position to form the fourth degree of the (La) minor scale.

Thus, the highest (Ré) (which defines the Sa-*grama* scale) is necessary for the dominant chord of the major mode; by contrast, the lowest (Ré) (which defines the Ma-*grama* scale) is necessary for the subdominant chord of the minor mode. The major mode is therefore a Sa-*grama*-ic scale, whilst the minor mode is a Ma-*grama*-ic scale.

3. Enharmonic positions used during modulations

If one modulates from the Zarlino scale situated on Do (or its relative minor scale, La) according to twelve ascending fifths and twelve descending fifths, one observes that it is necessary to have twelve additional tonal positions (*shruti-s*), eleven of which are situated one comma lower in relation to the lowest positions of the original model, and one is situated one comma higher than the highest.

The table on page 32 indicates the enharmonic positions of chromatic scales associated with various chords and tonalities. The median tonal positions, belonging to the cycle of descending fifths, are represented by the symbol "°". Positions raised or lowered by a comma are represented, respectively, by the symbols "+" and "—". The symbols in parentheses correspond to the "critical" notes of the Ma-*grama* and Sa-*grama* fundamental scales (see II.5).

In a group of three horizontal lines on the table, the name of the note that begins the median line marks the tonality on which the Zarlino scale is situated, the notes of its first and sixth degree chords, its relative minor scale, and the enharmonic structure of the chromatic scale that is "in the background". The horizontal lines immediately above and below a given line show the enharmonic positions of the other chords of the scale except that of the seventh degree.

The experiments that we have conducted with M. Pierre-Yves Asselin, performing on the *shruti harmonium* pieces by J.-S. Bach and other composers in pure intonation according to the principles outlined above, suggest that practical research in this domain could allow for a different hearing of harmonic music. Composers who work on computers would have every interest in using the analytical rules of the *grama-murchana* system to measure the effect in the contexts of classical and contemporary harmony.

GLOSSARY

antara gandhar: the major harmonic third of the fundamental scale (Ma-*grama* or Sa-*grama*).

aṣavari: mode of La.

bilaval: mode of Do.

bhairao: mode of Do with lowered second and sixth.

bhairavi: mode of Mi.

cyuta: lowered by a comma.

comma: 81/80.

demi-ton chromatique: 25/24.

Dha (*dhaivat*): sixth degree of the scale (La).

Ga (*gandhar*): third degree of the scale (Mi).

grama: fundamental scale defined in ancient Sanskrit treatises.

harmonique: set of tonal positions constructed from the harmonic major third or the harmonic minor sixth below the tonic.

kakali nishad: the harmonic major seventh (Si) of the fundamental scale (Ma-*grama* or Sa-*grama*).

kalyan: mode of Fa.

kaphi: mode of Ré.

khamaj: mode of Sol.

komal: flat or "minor".

lalita: mode of Si with the major third and major seventh.

limma: 256/243.

Ma (*madhyam*): fourth degree of the scale (Fa).

marva: mode of Fa with the minor second.

murchana: mode or modal transposition.

Ni (*nishad*): seventh degree of the scale (Si).

Pa (*pancham*): fifth degree of the scale (Sol).

purvi: mode of Fa with the minor second and sixth.

pythagoricien: set of tonal positions obtained by cycles of fifths or fourths.

raga: principle of musical organisation, at the base of Indian music. The ragas are distinguished by their tonal scales, their melodic movements and the notes "accented" or "muffled".

Re (*rishabh*): second degree of the scale (Ré).

sadharana: transposition of a comma.

Sa (*sadja*): tonic or first degree of the scale.

sandhiprakash: the change of light, that is to say dawn or dusk.

sangita: music.

shastra: written tradition.

shruti: 1. Potential position of a note in the octave. 2. Interval between two positions.

shuddha grama: fundamental scale of origin (without using altered notes — *vikrit swara-s*).

shuddha swara: "natural" notes. Note that *kaphi* is the ancient "natural" scale and *bilaval* the modern one.

sphota: clear and distinct manifestation.

swara: note.

tivra: sharp, or "major".

todi: mode of Mi but with the fourth and seventh augmented.

vadi swara: "tension" note of a given raga.

vikrit swara: "altered" note (ancient term). The "altered" notes are *antara gandhar* and *kakali nishad*.

SUMMARY

The theory of just intonation based on two fundamental scales (*grama-s*) described in Bharata's *Natya Shastra* and other early Sanskrit musicological texts formalises the relations found in diatonic musical scales with an astounding perspicuity. Only minimal changes to that system are necessary to account for the non-diatonic scales also popular in contemporary Indian music. The paper argues that Bharata's theory provides a better psycho-acoustic rationale for playing ragas at particular times of the day than the current explanation. The mathematical model of this system of interval relations in diatonic and related scales presented here is a useful tool to study the relations further. Discussion ends on a consideration of implications of the Indian system for harmonic music in just intonation.

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