



groupe
représentation
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CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE
31, chemin Joseph Aiguier
F-13402 MARSEILLE CEDEX 9 (France)

PATTERN GRAMMARS IN FORMAL REPRESENTATIONS OF MUSICAL STRUCTURES

Bernard Bel

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This paper introduces several formal models of pattern representation in music. *Polyvalent multimodal grammars* describe partially overlapping sound events as found in polyphonic structures. *Bol Processor grammars* are characterisations of sequential events in terms of substring repetitions, homomorphisms, etc. Parsing techniques, stochastic production and recent developments of BP grammars are briefly described.

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GRTC /352/ Juin 1988

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11th International Joint Conference on Artificial Intelligence, Workshop on AI & Music, 20 August 1989, pp.113-42

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Abstract

This paper introduces several formal models of pattern representation in music. *Polyvalent multimodal grammars* describe partially overlapping sound events as found in polyphonic structures. *Bol Processor grammars* are characterizations of sequential events in terms of substring repetitions, homomorphisms, etc. Parsing techniques, stochastic production and recent developments of BP grammars are briefly described.

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*La musique est le suprême
mystère des sciences de
l'homme, celui contre
lequel elles butent et qui
garde la clé de leur progrès.*

(Lévi-Strauss 1964:27)

Music representation is a vast subject covering two separate topics: sound models and musical structures. Although in the field of computer-aided composition much work has been devoted to new sound generation techniques, many studies of structural models are still limited to the analysis of Western tonal music. Beyond this domain (particularly in ethnomusicology) there has been a tendency to borrow concepts from linguistics on the basis of assumed parallels between language and traditional musical systems. Feld's provocative paper (1974), however, introduced to ethnomusicologists an epistemological dilemma which resulted in the virtual rejection of purely abstract speculations that Feld termed 'the hollow shell of formalism':

Only Blacking (...) and Lindblom and Sundberg (...) have dealt explicitly with basic theoretical issues... The rest of the literature ignores issues like the empirical comparison of models, a metatheory of music, evaluation procedures, and the relation of the models to the phenomena they supposedly explain. (Feld 1974:210).

'Precomputer' studies have been limited to descriptions of 'frozen' data, thereby ignoring models that would produce musical scores:

¹ Bernard Bel is a computer scientist with a background in electronics. Since 1979 he has been collaborating with anthropologists, musicologists and musicians on a scientific study of North Indian melodic and rhythmic systems. In 1981 he built a sophisticated real-time *Melodic Movement Analyser* (MMA) with the aid of which he developed automatic transcription and analysis of raga music (in collaboration with Dr. Wim van der Meer). In 1986 he joined GRTC, an AI laboratory of the *Centre National de la Recherche Scientifique* in Marseille. At present he is working on algorithms for automatic rule generation derived from hypotheses on training methods in traditional *tabla* (in collaboration with ethnomusicologist Dr. Jim Kippen). He is also involved in research on computer music at the *Laboratoire Musique et Informatique de Marseille* (MIM).

Our theory of music is...based on structural considerations; it reflects the importance of structure by concerning itself not with the composition of pieces but with assigning structures to already existing pieces.

(Jackendoff & Lerdahl 1982:85)

In fact, computers make it possible not only to check analytical models against large data sample sets but also to adjust grammars with a view to improving the machine's ability to compose music or to assist a human composer by suggesting channels of 'musical thinking' unknown to him/her. This is — broadly speaking — the 'expert system' approach (Kippen & Bel 1989c)². Several teams are working on Western tonal music along these lines, trying to set-up validations of the structural models proposed by Ruwet and Jackendoff & Lerdahl (Camilleri 1989).

Research in analysis and composition of music by means of computers implies the choice of adequate representational models of music. This paper deals with issues relevant to both approaches. Two models of musical patterns imbedded in formal grammars are introduced: *Polymodal Multivalent Grammars* (PMG) and *Bol Processor* (BP) grammars. PMGs have been proposed by Vecchione (1978ff) as a general syntactic model of music which lends itself to a paradigmatic analysis based on *resemblance*, and may therefore be used as an aid to music composition³. BP grammars have been designed by Kippen & Bel (1984) for an experimental study of improvisation/evaluation schemata in North Indian *tabla* drumming. The Bol Processor was first implemented in assembly language on the Apple IIc for fieldwork with traditional musicians⁴. Recent developments of this model in the light of contemporary music theory and practice are briefly introduced in §2.5⁵.

A detailed description of the BP's modus operandi is found in Kippen & Bel (1989a). The problem of transferring knowledge from informants to machines using automatic rule generation — the present focus of the author's project — is discussed in Kippen & Bel (1989b,c,d).

1. Pattern grammars, languages, and musical structures

1.1. Definitions and notations

It is assumed that elementary musical events ('atoms') may be transcribed with the aid of a finite set of symbols, namely the *alphabet* V_t of *terminal symbols*. For example, many pieces of *tabla* music belonging to the *qa'ida* repertoire may be transliterated with $V_t = \{dhin, tin, dhe, tee, dha, ta, ghi, ki, ge, ke, ka, na, ne, ra, -\}$, in which the hyphen denotes a silence⁶. Each symbol is a quasi-onomatopoeic mnemonic representation of a musical event (stroke or

² Another approach, which will not be discussed here, is the discovery of a system for music composition or improvisation from musical scores (Laske 1984, Kippen & Bel 1989b).

³ Vecchione's model takes in consideration polyphonic time structures and other features like modes of space location, modes of significance, modes of voice/instrument processing, modes of performance, etc.

⁴ Initial work was part of a research scheme by the *International Society for Traditional Arts Research* (ISTAR, New Delhi) generously funded by the *Ford Foundation* and the *National Centre for the Performing Arts* (NCPA, Bombay). Subsequently, Kippen's work was also supported by the *Leverhulme Trust* in the UK.

⁵ An outcome of discussions in the 'OC' study group involving composers and scientists at *Laboratoire Musique et Informatique* de Marseille (MIM).

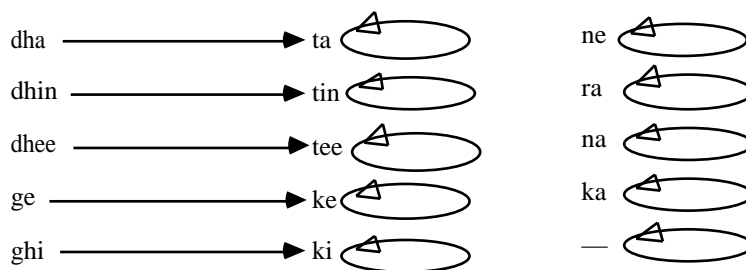
⁶ A system of transliterating *tabla* strokes to non-ambiguous symbolic representations has been proposed by Kippen (1988:xvi-xxiii).

*bol*⁷). In the same way, any finite set of elementary sound events (e.g. notes) may be viewed as a terminal alphabet for music.

A *monodic sequence* is a string on V_t . The set of finite strings on alphabet V_t is notated V_t^* . The length of string s is $|s|$. For instance, $|dhee na ge na| = 4$. The nil string is λ , hence $|\lambda| = 0$. The set on non-nil finite strings on V_t is V_t^+ . In set notation, $V_t^+ = V_t^* - \{\lambda\}$. The number of elements in a set S (the *cardinal* of S) is notated $\text{card}(S)$. \emptyset denotes the empty set, hence $\text{card}(\emptyset) = 0$.

To describe languages we may need an enumerable set of *variables* or *non-terminal* symbols V_n such that $V_n \cap V_t = \emptyset$. Throughout this paper, variables will be represented with upper-case characters sometimes followed with numbers denoting paradigmatic variants or metric durations, e.g. X2, TA14, etc...

Tabla strokes are either *open* (voiced) or *closed* (unvoiced). Any sequence of unvoiced strokes may be seen as the closed (*mirror*) image of a sequence of open strokes through a mapping of V_t^* to itself. This mapping *mir* is a λ -free homomorphism of V_t^* to itself whose reduction to V_t is an idempotent mapping of V_t to itself, for instance:



ta is notated as $\text{mir}(dha)$, and *teenakena* as $\text{mir}(dheenagena)$.

Homomorphisms are suitable for describing any structural relationship that is built on transformations of the terminal symbols, e.g. transposition, octave shift, etc. in tonal music.

1.2. Pattern grammars

Leaving aside its parallels with language, music may be seen as an algorithmic (non-random) signal sequence. Since automatic (i.e. automata-generated) sequences lie somewhere between periodicity and chaos, yet closer to periodicity (Allouche 1987:264), a workable hypothesis is one that envisages the description of all kinds of musical events as generated by automata, or, equivalently, formal grammars.

Yet both the atomistic theory of music — “la mélodie n’est autre chose qu’une succession de sons déterminés” (Vincent d’Indy) — and the classical ‘scheme’ model — “ABA is a Lied pattern” — are one-level conceptualizations of music. The atomistic conception, which was followed by Schönberg, Webern, etc.⁸, resulted in attempts to proclaim *set theory* as the foundation of tonal (cf. Babbitt) and atonal (cf. Forte) music. As put by Roads (1984:10):

⁷ from the verb *bolna*, ‘to speak’, in North Indian languages; for this reason the machine was named ‘Bol Processor’.

⁸ later by followers of the stochastic approach: Betty Shannon (1951), Meyer (1956-7), Coons & Kraehenbuehl (1958), Fuchs (1961), Moles (1958), Xenakis (1963), Philippot (1970)

With this approach, abstract properties — such as invariance relationships under formal operations — may be studied, while properties which are not amenable to such an approach are often ignored (e.g. timbral and spatial relationships, articulation factors, clusters and clouds of sound, dramatic and expressive qualities, and so on).

Ruwet (1966,1972:100-34) and Nattiez (1976:239-78) — both involved in surface, Schenkerian and prosodic analysis — and later Jackendoff & Lerdahl (1983)⁹, introduced linguistic/semiotic descriptions in which several layers of music constituents may be assigned to a single (monodic or homophonic) sequence.

Unfortunately, these sequential models have been unable to account for the structural organization of overlapping constituents as found in polyphonic musical structures:

At the present stage of development of the theory, we are treating all music as essentially homophonic (...) For the more contrapuntal varieties of tonal music, where this condition does not obtain, our theory is inadequate. We consider an extension of the theory to account for polyphonic music to be of great importance. However, we will not attempt to treat such music here except by approximation. (Lerdahl & Jackendoff 1983:37)

To achieve this, it is necessary to disconnect the set of *musical constituents* from the set of the *time-span intervals* which support them¹⁰. The set of constituents is always structured as a *tree* (a hierarchy of partitions), whereas the set of time-span intervals is structured as an *join-semilattice* (Birkhoff 1984:22) which may or may not be tree-shaped. Musical structure is therefore determined by a morphism between these two structures: a bijective morphism defines a sequential structure whereas injective morphisms define non-sequential structures (Vecchione 1984:139ff, 1985:207ff).

The idea of dissociating musical constituents from their time-span intervals comes in response to Xenakis' (1963, 1972:57) suggestion to separate *out-time structures*, the *structure of time* and the resulting *in-time structure*, i.e. the Cartesian product of the former two.

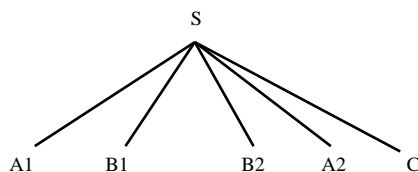
Consider for instance the 9th measure (2nd theme of the first movement) in J.S. Bach's fifth *Brandenburg Concerto* in D major:

⁹ a model recognising four components: metrical structure, grouping structure, time-span reduction and Schenkerian prolongational reduction — the latter was criticised and expanded by Deliège (1983-4).

¹⁰ This hypothesis has not been considered by all researchers in the field. See for instance Chemillier (1986:2, 1987,2:380ff). Interestingly, Xenakis introduced three structures in his early work (1963:190-1,200): in-time, out-time structures and structure of time, the former being a mapping between the latter two. Yet in 1972 he did not deal any more with the structure of time.

(Vecchione 1984:137)

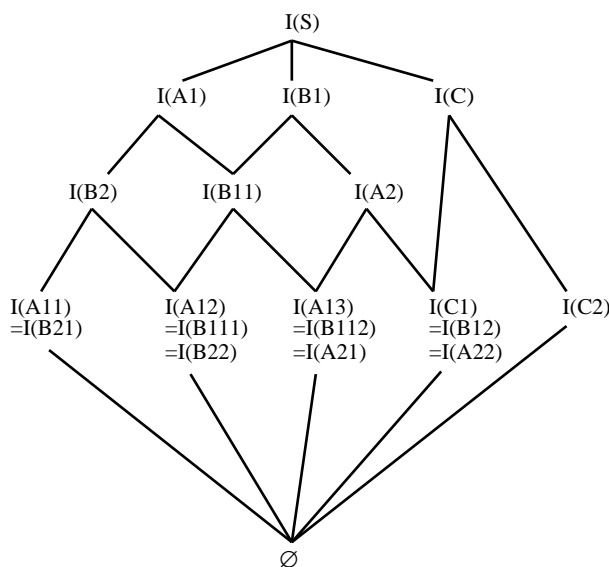
The set of constituents may be structured as a tree



corresponding to the following time-span intervals

I(A1)			I(C)	
I(A11)	I(A12)	I(A13)	I(C1)	I(C2)
	I(B1)			
	I(B11)		I(B12)	
	I(B111)	I(B112)		
I(B21)	I(B22)	I(A21)	I(A22)	
I(B2)		I(A2)		

which are structured as a lattice:



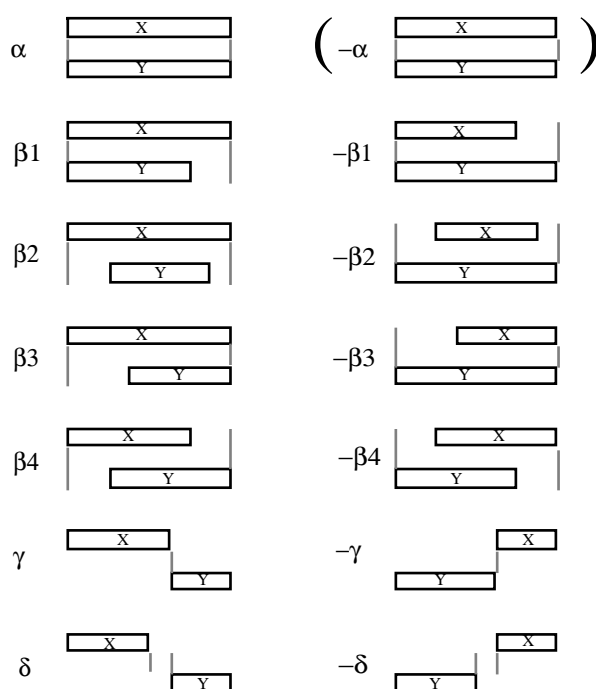
Note that if the universal lower bound '∅' is not represented, the structure becomes a join-semilattice with universal upper bound I(S), the *minimal elements* of which are arranged in a strict sequence: I(A11)=I(B21), I(A12)=I(B111)=I(B22), I(A13)=..., I(C2).

The weakness of this representation lies in the fact that some of the time-span intervals have been chopped into smaller intervals, e.g. I(B11), I(B12) that do not necessarily support meaningful musical constituents. The same problem arises in set representations, e.g. Chemillier & Timis (1988) who developed a formal language for polyphonic music based on

trace languages and *partially commutative free monoids* (Mazurkiewicz 1984, Chemillier 1987).

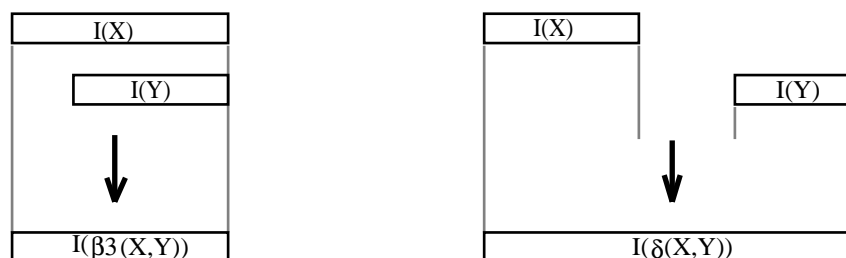
Another aspect of the present research, therefore, is an attempt to deal with time as a *topological* entity: considering non-Euclidian metric durations is necessary when dealing with contemporary music, most extra-European systems, and European music prior to the 15th Century. Wiggins & al. (1988) pursued this line of thinking in a logic representation of musical constituents, defining an ordered set of ‘times’ and a linearly ordered commutative group ‘duration’. Another fruitful approach is based on a single set of open intervals (of rational numbers) carrying two binary relations: inclusion and precedence (Van Benthem 1983:58-79).

In the linguistic/semiotic model, applying a rule in a formal grammar amounts to a step in string *concatenation*. In the description of polyphony, however, thirteen distinct operations are possible (Van Benthem 1984:70, Vecchione 1984:150ff):



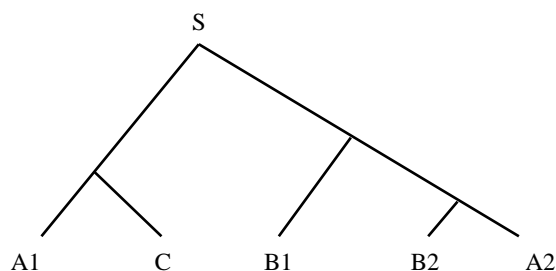
Comparisons of time-span intervals may be worked out by calculus or with reference to the fact that that the ear may perceive event X as starting or ending before, simultaneously, or after event Y.

If $I(X)$ and $I(Y)$ are the time-span intervals supporting X and Y respectively, operations α , β_1 , ..., δ yield a constituent $X \cup Y$, the time-span interval of which is the smallest interval covering both $I(X)$ and $I(Y)$, e.g.



Note that γ is a strict sequence and both α and $-\alpha$ strict superimpositions. The latter two are identical.

If a binary-tree structure is assigned to constituents A1, A2, B1, B2 and C in the Bach example, e.g.



then S is the result of the following operation:

$$S = \beta_1(\gamma(A1,C), -\beta_3(B1, \gamma(B2,A2)))$$

Obviously this description does not retain the entire topological information: the combination of A1 and B1 might as well have been $\beta_4(A1,B1)$ or $\beta_2(A1,B1)$.

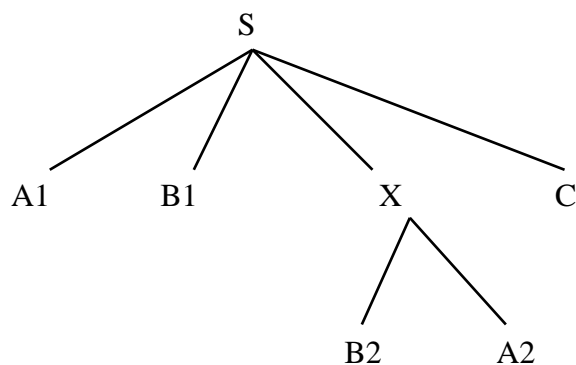
If more information must be made explicit, then alternative formulae, built on all the supposingly ‘meaningful’ binary trees — in terms of musical analysis — may reduce ambiguity:

$$S = \beta_1(\gamma(A1,C), -\beta_3(B1, \gamma(B2,A2))) = -\beta_4(\alpha(\beta_4(A1,B1), \gamma(B2,A2)), C) = \dots \text{ (up to } 5! = 120 \text{ formulae).}$$

Vecchione (1984:185ff) implicitly used time operators as *predicates* (i.e. Boolean operators) in his syntactic model (‘ Ω -syntax’), actually using the same notation for operators and their corresponding predicates. In this way, the first expression of S may be written:

$$\begin{aligned} S &\longrightarrow \beta_1(X,Y) \\ X &\longrightarrow \gamma(A1,C) \\ Y &\longrightarrow -\beta_3(B1,Z) \\ Z &\longrightarrow \gamma(B2,A2) \end{aligned}$$

Rules in this format may contain more than one predicate, thereby allowing hierarchical descriptions more general than binary trees, e.g.

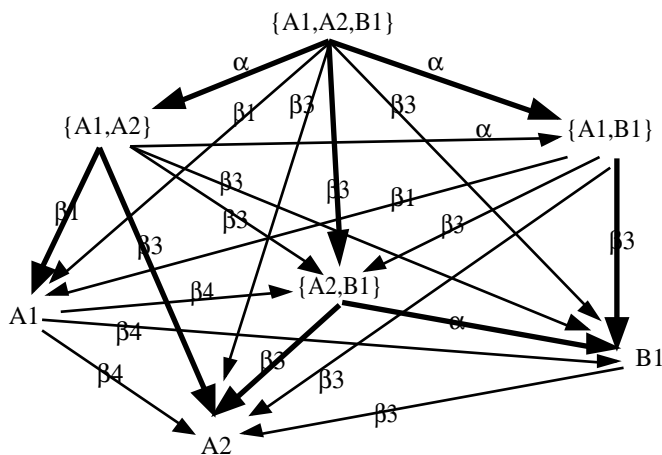


resulting in a more compact (and less ambiguous) representation:

$$\begin{aligned}
 S &\longrightarrow \beta_4(A1,B1) \wedge \beta_1(A1,X) \wedge \gamma(A1,C) \wedge \beta_3(B1,X) \wedge \beta_4(B1,C) \wedge \beta_4(X,C) \\
 X &\longrightarrow \gamma(B2,A2)
 \end{aligned}$$

where the right side of the first rule is a conjunction of predicates¹¹.

In many cases only part of the information on time-span intervals is musically significant. If $E = \{A1,A2,B1,B2,C\}$, $P(E)$, the set of all subsets of E , is a complete lattice which may be represented as a graph in which each node indicates a subset. Edges marking the inclusion relation $X \supset Y$ may be labeled with the predicate which is verified on (X,Y) , in fact always α , β_1 , or β_3 . The graph may then be complemented until it is complete. Other temporal relations: β_2 , β_3 , β_4 , γ , δ (and their symmetric counterparts) are visible in the complete graph. Consider for instance the subgraph concerning $A1$, $A2$ and $B1$ in the example:



Bold edges are those belonging to the lattice.

If only part of the information about time is known (i.e. significant), it may be completed with the aid of a *transitivity table*: given $f(X,Y)$ and $g(Y,Z)$, the table yields $f \circ g(X,Z)$. In many cases the result is a disjunction of predicates; in some cases it is even unknown (hence it is a disjunction of the thirteen predicates). Part of the 13x13 square table is:

¹¹ These examples of tree structures do not claim any musical relevance.

\nearrow *	$-\beta_2$	β_2	β_4	γ	δ
$-\beta_2$	$-\beta_2$	$\alpha \vee \beta_1 \vee \beta_2 \dots$ (unknown)	$-\beta_1 \vee -\beta_2 \vee \beta_4$ $\vee \gamma \vee \delta$	δ	δ
β_2	$\alpha \vee \beta_1 \vee -\beta_1 \vee \beta_2$ $\vee -\beta_2 \vee \beta_3 \vee -\beta_3$ $\vee \beta_4 \vee -\beta_4$	β_2	$\beta_2 \vee \beta_3 \vee \beta_4$	$\beta_2 \vee \beta_3 \vee \beta_4$	$\beta_2 \vee \beta_3 \vee \beta_4 \vee \gamma \vee \delta$
β_4	$-\beta_1 \vee -\beta_2 \vee \beta_4$	$\beta_2 \vee \beta_3 \vee \beta_4 \vee \gamma \vee \delta$	$\beta_4 \vee \gamma \vee \delta$	δ	δ
γ	$-\beta_1 \vee -\beta_2 \vee \beta_4$	δ	δ	δ	δ
δ	$-\beta_1 \vee -\beta_2 \vee \beta_4$ $\vee \gamma \vee \delta$	δ	δ	δ	δ

$$\mathbf{f} * \mathbf{g} \mid \mathbf{f}(\mathbf{X}, \mathbf{Y}) \wedge \mathbf{g}(\mathbf{Y}, \mathbf{Z}) \longrightarrow \mathbf{f} * \mathbf{g}(\mathbf{X}, \mathbf{Z})$$

For example, $\beta_2(\mathbf{X}, \mathbf{Y}) \wedge \gamma(\mathbf{Y}, \mathbf{Z}) \longrightarrow [\beta_2 \vee \beta_3 \vee \beta_4](\mathbf{X}, \mathbf{Z})$. The graph with incomplete information about time may be computed as a *constraint network* (Allen & Kautz 1985:255ff). The model makes no assumptions on relations that have deliberately been left unknown. Rather it is an enumeration of all structures consistent with the input data.

Unambiguous predicate representation requires $n(n-1)/2$ predicates in a rule containing n variables. The following matrix lay out is self-explanatory:

	A1	B1	X	C
A1	α	β_4	$-\beta_1$	γ
B1	$-\beta_4$	α	$-\beta_3$	$-\beta_4$
X	β_1	β_3	α	$-\beta_4$
C	$-\gamma$	β_4	β_4	α

$$S \longrightarrow \begin{pmatrix} \alpha & \beta_4 & -\beta_1 & \gamma \\ -\beta_4 & \alpha & -\beta_3 & -\beta_4 \\ \beta_1 & \beta_3 & \alpha & -\beta_4 \\ -\gamma & \beta_4 & \beta_4 & \alpha \end{pmatrix} [A1, B1, X, C]$$

or equivalently:

$$S \longrightarrow T [A1, B1, X, C] \quad \text{with} \quad T = \begin{pmatrix} \alpha & \beta_4 & -\beta_1 & \gamma \\ -\beta_4 & \alpha & -\beta_3 & -\beta_4 \\ \beta_1 & \beta_3 & \alpha & -\beta_4 \\ -\gamma & \beta_4 & \beta_4 & \alpha \end{pmatrix}$$

Vecchione's general format for context-sensitive *time-predicate rules* (which he calls ' Ω -syntax rules') is the following (1985:276):

$$\theta_q [A J B] \longrightarrow \theta_r [C K D]$$

in which θ_q , θ_r are matrixes of time predicates, and A, B, C, D possible contexts. A, B, C, D, J, K are strings of variables corresponding to 'significant' units¹². In general, $|K| \geq |J|$ to ensure decidability. If A=C and B=D the rule is context-sensitive in the common sense. The following subclass of rules

$$\theta_q [C J D] \longrightarrow \theta_q [E J F]$$

may be called *change of context*. Depending on whether changes occur in the time-predicate matrix, in the medium part of the list ('J') or in its right and left contexts, eight modes of transformation can be defined and related to musical transformations¹³ (Vecchione 1985:277ff).

In grammars, unary or n -ary predicates may also be used to express relationships other than those on time-span intervals: location in time and space, modes of significance, voice or instrument processing, etc. Grammars containing several classes of predicates (taking their arguments in several sets of values) have been called *Polymodal Multivalent Grammars* (PMG) (Vecchione 1985:257ff).

Time predicates are equivalent to the thirteen primitive relations in Allen & Kautz's model of naive temporal reasoning (1985), while Oppo (1984) considered only seven relations given the fact that, for any time predicate ω , $\omega(X,Y) = -\omega(Y,X)$. In a PMG it is necessary to use the thirteen values to obtain a canonic rule format that is meaningful for computing rule resemblances (Bel & Vecchione 1989).

Formulae using time operators and conjunctive expressions of time predicates may be matched or evaluated using tree unification as implemented in the Prolog language (Colmerauer 1984). The Prolog II-III implementation of generation/parsing techniques for PMGs is under study at the MIM laboratory (Risch 1988).

PMGs belong to a class of formal grammars which we call *pattern grammars*:

| A pattern grammar is an ordered 6-tuple (V_t, V_n, A, S, F, P) such that:
 | V_t is the terminal alphabet, V_n a finite set of variables, $V_t \cap V_n = \emptyset$ and $S \in V_n$;
 | A is any enumerable set;
 | P is a set of predicates, i.e. applications of $(V_n \cup A)^n$ to $\{\text{true, false}\}$ where n is a positive integer;
 | F is a finite number of triplets (p,q,R) such that $p,q \in (V_n \cup V_t)^+$ and R a conjunctive expression using predicates in P whose arguments are variables occurring in q .

Triplets of F can be notated: $p \longrightarrow q$ where R . In many cases, A is the set of positive integers; consequently, predicates may accept numeric arguments, see for instance §2.1.

¹² Derived from a meaningful segmentation.

¹³ This should only be regarded as an initial step towards a theory of musical transformations.

In the field of pattern recognition, Evans (1969) first proposed an effective procedure for inferring pattern grammars from examples. This author has achieved a similar goal using top-down parsing and postponed unification in a Prolog II program (Bel 1988).

The pattern grammar concept may be further elaborated depending on its actual implementation (rewriting systems, logic, etc.) Contextual information may be conveyed in a sophisticated way, e.g. *feature-grammars* or *metamorphosis grammars* (Colmerauer 1978). Metamorphosis grammars are easier to parse than context-sensitive grammars (Véronis 1986). An example of logic representation implicitly conveying contextual information may be found in Wiggins & al. (1988).

We shall see (§2.1) that *Bol Processor grammars* are a combination of pattern grammars and a sequential model derived from *pattern languages*. The following (§1.3—1.4) is a formal description of the sequential model which may be skipped by readers unfamiliar with formal language theory.

1.3. Pattern languages

If H denotes the set of all λ -free homomorphisms (with respect to concatenation) of $(V_n \cup V_t)^*$ to itself, an element of H which is the identity when restricted to V_t is called a *substitution*. If the image set of a substitution is V_t^* , the substitution is *terminal*. A substitution which is a bijection of V_n to itself when restricted to V_n is called a *renaming of variables*.

A *pattern* is any element of $(V_n \cup V_t)^*$. If p is a pattern and s a substitution, then $s(p)$ is a *derivation* of p . A pattern containing no variable is a *terminal derivation* or a *sentence*. Two patterns, p and q , are *equivalent* (i.e. $p \approx q$) if and only if there exists a renaming of variables r such that $p = r(q)$. Another binary relation (notated $p \leq q$) is defined as follows: p is *less general than* (or *more specific than*) q iff for some substitution s , $p = s(q)$. Since a substitution is a λ -free homomorphism, $p \leq q \Rightarrow |p| \geq |q|$.

The *language* generated by a pattern p is the set of terminal derivations of p , namely $L(p) = \{s \in V_t^+ : s \leq p\}$. It can be proved that:

\leq is transitive

$p \leq q \Rightarrow L(q) \supseteq L(p)$

$p \approx q \Leftrightarrow p \leq q$ and $q \leq p$

However, the question of finding an effective procedure to decide whether $L(q) \supseteq L(p)$ given arbitrary patterns p and q appears to be open (Angluin 1980:49-52).

The mirror mapping (see 1.1) can be extended to the set of patterns with the following definition:

$\forall p, q \in (V_n \cup V_t)^*, p = \text{mir}(q) \Leftrightarrow \forall s \in V_t^+, s \leq q \Rightarrow \text{mir}(s) \leq p$

A transcription of a few variations of a *qa'ida* is given below¹⁴:

[1]	dhin--dhagena tagetirakita	dha--dhagena dhin--dhagena	dhatigegegenaka dhatigegegenaka	dheenedheenagena teeneteenakena
	tin--takena tagetirakita	ta--takena dhin--dhagena	tatikekenaka dhatigegegenaka	teeneteenakena dheenedheenagena

¹⁴ *Qa'ida* is an Urdu word meaning 'rules' or 'system', and is derived from the Arabic word *qava'id* meaning 'grammar'. Tabla players often refer to the concept of grammaticality to denote correct arrangements of 'words' built on a limited vocabulary of *bols*.

[2]	dhin--dhatige tagetirakita tin--tatike tagetirakita	genakadhin-- dhin--dhagena kenakatin-- dhin--dhagena	tirakitatira dhatigegegenaka tirakitatira dhatigegegenaka	kitatirakita teeneteenakena kitatirakita dheenedheenagena
[3]	dhin--dhagena tagetirakita dhin--dhagena tagetirakita tin--takena taketirakita dhin--dhagena tagetirakita	dha--dhagena dhin--dhagena dha-dha-dha- dhin--dhagena ta--takena tin--takena dha-dha-dha- dhin--dhagena	dhatigegegenaka dhatigegegenaka dhatigegegenaka dhatigegegenaka tatikeyakakana tatikeyakakana dhatigegegenaka dhatigegegenaka	dheenedheenagena teeneteenakena teeneteenakena teeneteenakena teeneteenakena teeneteenakena teeneteenakena dheenedheenagena
[4]	dhin--dhagena tagetirakita dheenedheenagena tagetirakita tin--takena taketirakita dheenedheenagena tagetirakita	dha--dhagena dhin--dhagena dheenedha-dheene dhin--dhagena ta--takena tin--takena dheenedha-dheene dhin--dhagena	dhatigegegenaka dhatigegegenaka dhatigegegenaka dhatigegegenaka tatikeyakakana tatikeyakakana dhatigegegenaka dhatigegegenaka	dheenedheenagena teeneteenakena teeneteenakena teeneteenakena teeneteenakena teeneteenakena teeneteenakena dheenedheenagena

Tabulation has been used to denote beats; each beat comprises six strokes (*bols*)¹⁵.

Conventionally, the first item is the *theme* of the *qa'ida* whereas [2], [3] and [4] are called *variations*. The problem may be defined as the description of the whole set of possible variations of this theme. One obvious feature of this sample sequence is that some variations are of the same duration as the theme (16 beats) while others are double (32 beats). Doubling the length is an improvisation technique commonly used by tabla players of the Lucknow style (Kippen 1988:166-67). These sets of variations may be roughly described with the following patterns, where X, Y, Z are variables:

Single variations:

p1 = X tagetirakitadhin--dhagenadhatigegegenakateeneteenakena
Y tagetirakitadhin--dhagenadhatigegegenakadheenedheenagena

Double variations:

p2 = dhin--dhagenadha--dhagenadhatigegegenakadheenedheenagena
tagetirakitadhin--dhagenadhatigegegenakateeneteenakena
Z dhatigegegenakateeneteenakena
tagetirakitadhin--dhagenadhatigegegenakateeneteenakena
tin--takenata--takenatatikeyakakateeneteenakena
taketirakitatin--takenatatikeyakakateeneteenakena
Z dhatigegegenakateeneteenakena
tagetirakitadhin--dhagenadhatigegegenakadheenedheenagena

Let L be the language representing all acceptable variations of this *qa'ida*, L1 the subset of single variations, and L2 the subset of double variations: $L = L1 \cup L2$. L(p1) and L(p2) are the languages generated by p1 and p2 respectively. We will agree (on the basis of much larger sample sets) that L(p1) and L(p2) contain non trivial subsets of L1 and L2. An elaborated description of L1 and L2 has been published in Bel (1987b).

¹⁵ In the onomatopoeic system used by North Indian drummers, Vt is practically a *prefix* code mapping strokes/sounds to the English lower-case alphabet. In other words, any string of Vt* (like *dheenedheenagena*) can be identified deterministically from left to right as a unique sequence of *bols* (*dhee/ne/dhee/na/ge/na*). Therefore strings of Vt* are usually represented without spaces.

Ideally, the search is for *descriptive* patterns of L1 and L2. A pattern d is descriptive of a language L if $L(d) \supseteq L$ and for every pattern q such that $L(q) \supseteq L$, $L(q)$ is not a proper subset of $L(d)$.

L is always a finite set: in this *qaida*, for example, all acceptable strings of Vt^* have lengths shorter than or equal to $32 \times 6 = 192$ symbols, therefore $\text{card}(L) \leq (1 + \text{card}(Vt))^{192}$. Yet, all we know about L is a set of positive and negative instances (correct and incorrect variations) called the *presentation*. The subsets of positive and negative instances are notated S^+ and S^- .

A pattern p *matches* a presentation $S = S^+ \cup S^-$ iff p is descriptive of S^+ and $S^- \cap L(p) = \emptyset$. A pattern p is a *tight fit* of S if $L(p) = S^+$. Evidently, any tight fit of S is descriptive of S^+ and matches S .

There is an effective procedure to infer a descriptive pattern from a set of positive instances S^+ , but the problem appears to be NP-complete in most cases (Angluin 1980:54-55). In addition, the class of pattern languages is not closed under any of the basic set operations: union, complement, and intersection. Therefore it is not possible to construct systematically a pattern description of a language. In order to achieve such an analytical task it is necessary to define more suitable classes of languages.

1.3. Restricted pattern languages (RPL)

If p is a pattern, a subclass of $L(p)$ may be defined with the aid of restrictions on acceptable derivations of p . For example, we may define the subclass of strings in $L(p)$ with specified length n : $L_n(p) = \{s \in Vt^+ : s \leq p \text{ and } |s| = n\}$. In the example above, evidently $|X| = |Y| = 24$ and $|Z| = 12$. In addition, we can write that $Y = \text{mir}(X)$.

There are other ways to formalize specifications with the aid of *rewriting rules*: we notate $p \longrightarrow q$, the fact that *every acceptable derivation of q is an acceptable derivation of p* , i.e. $q \leq p$. If \mathcal{S} is the set of all substitutions, $p \longrightarrow q$ denotes the subset $\mathcal{S}_{p \rightarrow q}$ such that $\forall s \in \mathcal{S}_{p \rightarrow q}, s(p) = q$. Since the problem of deciding whether $p \leq q$ given arbitrary patterns p and q is NP-complete (Angluin 1980:52), we must restrict to *well-formed rules*:

Definition:

Rule $p \longrightarrow q$ is well-formed iff $p \in Vn^+, q \in (Vt \cup Vn)^+, |q| \geq |p|$, and no variable occurs twice in p .

Theorem:

If rule $p \longrightarrow q$ is well-formed then $L(p) \supseteq L(q)$.

Proof:

All we need to find is a substitution s such that $q = s(p)$ that yields $q \leq p$. A procedure for finding s is the following:

- (1) substitute the $|p|-1$ leftmost variables in p with the $|p|-1$ leftmost symbols in q
 - (2) substitute the rightmost variable in p with the string made of the $|q|-|p|+1$ rightmost symbols in q .
- Since no variable occurs twice in p , there are no restrictions on q that render substitutions impossible. This procedure may in fact be derived from Makanin's algorithm (see Makanin 1977 or Roussel 1987). For example, a substitution of XYZ yielding $abcX$ is $\{X/a, Y/b, Z/cX\}$.

Let $L(p)$ be a pattern language and R a set of well-formed rewriting rules. Every rule ($p \longrightarrow q$) defines a set of substitutions $\mathcal{S}_{p \rightarrow q}$. To construct a sentence of the *restricted pattern language* $L_R(p)$ we proceed as follows:

- (a) select a subset R_0 of the set of rules R .
- (b) Let $\mathcal{S}_0 = \bigcap_i (\mathcal{S}_{p_i \rightarrow q_i})$ such that $(p_i \longrightarrow q_i) \in R_0$.
- (c) If $\mathcal{S}_0 \neq \emptyset$ and \mathcal{S}_0 is finite, then $\mathcal{S}_0 = \{s\}$ such that the sentence is $w = s(p)$.

The selection of R_0 requires the comments:

- (1) If a variable X does not appear in any premise of the selected set of rules, then it may be substituted to any $q \in (Vt \cup Vn)^+$. Consequently, S_0 is infinite and $L_{R(p)} = \emptyset$.
- (2) There are subsets of R that yield $S_0 = \emptyset$, i.e. *aborted derivations*. For example, if a variable X appears as the premise of two distinct rules in R_0 , $X \rightarrow q_i$ and $X \rightarrow q_j$, then $S_{X \rightarrow q_i} \cap S_{X \rightarrow q_j} = \emptyset$, hence $S_0 = \emptyset$.
- (3) In cases other than (1) and (2) every variable occurs in a pattern with a unique derivation, so that $\text{card}(S_0) = 1$.

The following example will clarify points (2) and (3). A RPL for double-length variations would be $L_R(p_2)$ where:

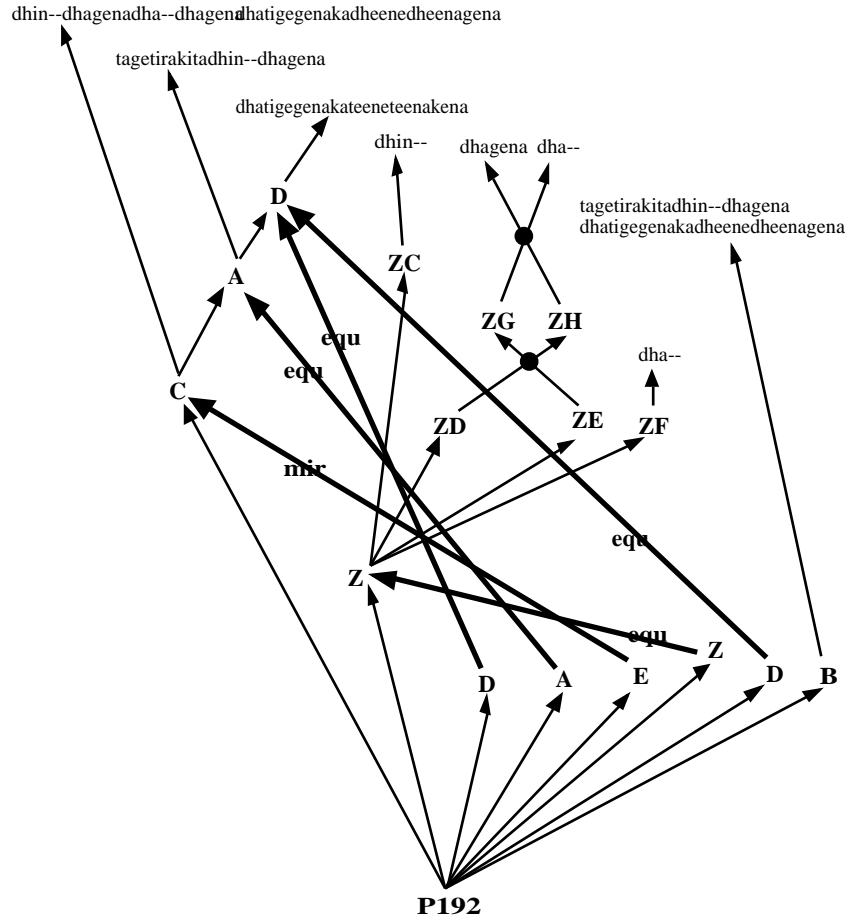
$p_2 = P192$ and R is the following set of rules:

- (1) $P192 \rightarrow C Z D A E Z D B$ where $E = \text{mir}(C)$
- (2) $A \rightarrow \text{tagetirakitadhin--dhagena D}$
- (3) $B \rightarrow \text{tagetirakitadhin--dhagenadhatigegegenakadheenedheenagena}$
- (4) $C \rightarrow \text{dhin--dhagenadha--dhagenadhatigegegenakadheenedheenagena A}$
- (5) $D \rightarrow \text{dhatigegegenakateeneteenakena}$
- (6) $Z \rightarrow \text{tirakitatirakitatirakita}$ (7) $Z \rightarrow ZA ZB$ (8) $Z \rightarrow ZC ZD ZE ZF$
- (9) $ZC ZD \rightarrow ZG ZH$ (10) $ZD ZE \rightarrow ZG ZH$ (11) $ZE ZF \rightarrow ZG ZH$
- (12) $ZG ZH \rightarrow \text{dhagenadhin--}$ (13) $ZG ZH \rightarrow \text{dhagenadha--}$
- (14) $ZC \rightarrow \text{dhin--}$ (15) $ZD \rightarrow \text{dhin--}$ (16) $ZE \rightarrow \text{dhin--}$ (17) $ZF \rightarrow \text{dhin--}$
- (18) $ZC \rightarrow \text{dha--}$ (19) $ZD \rightarrow \text{dha--}$ (20) $ZE \rightarrow \text{dha--}$ (21) $ZF \rightarrow \text{dha--}$
- (22) $ZA \rightarrow \text{dheenedheenedheene}$ (23) $ZB \rightarrow \text{dheenedheenedheene}$
- (24) $ZA \rightarrow \text{dha-dha-dha-}$ (25) $ZB \rightarrow \text{dha-dha-dha-}$

If we select $R_0 = \{1,2,3,4,5,8,10,13,14,21\}$, we obtain the sentence:

dhin--dhagena	dha--dhagena	dhatigegegenaka	dheenedheenagena	C
tagetirakita	dhin--dhagena	dhatigegegenaka	teeneteenakena	
dhin-- dhagena	dha--dha--	dhatigegegenaka	teeneteenakena	Z D
tagetirakita	dhin--dhagena	dhatigegegenaka	teeneteenakena	A
tin--takena	ta--takena	tatikekenaka	teeneteenakena	E
taketirakita	tin--takena	tatikekenaka	teeneteenakena	
dhin--dhagena	dha--dha--	dhatigegegenaka	teeneteenakena	Z D
tagetirakita	dhin--dhagena	dhatigegegenaka	dheenedheenagena	B

which may be represented with the following context-sensitive syntactic graph, where ‘equ’ and ‘mir’ indicate repetition and mirror structures:



Sets of substitutions for rules 10 and 14 may be represented as $\mathcal{S}_{p_{10} \rightarrow q_{10}} = \{\dots, ZD ZE/ZG ZH, \dots\}$ and $\mathcal{S}_{p_{14} \rightarrow q_{14}} = \{\dots, ZC/dhin-- , \dots\}$, where ‘...’ indicates the (denumerable) possible substitutions of all strings of $(V_t \cup V_n)^+$ except the specified ones (ZD ZE and ZC). We find that $\mathcal{S}_{p_{10} \rightarrow q_{10}} \cap \mathcal{S}_{p_{14} \rightarrow q_{14}} = \{\dots, ZD ZE/ZG ZH, \dots, ZC/dhin-- , \dots\}$.

The result of all intersections is:

$$\mathcal{S}_0 = \{P192/C Z D A E Z D B, A/tagetirakitadhin--dhagena D, B/tagetirakitadhin--dhagenadhatigenakadheenedheenagena, C/dhin--dhagenadha--dhagenadhatigenakadheenedheenagena A, D/dhatigenakateeneteenakena, Z/ZC ZD ZE ZF, ZC/dhin--, ZD ZE/ZG ZH, ZF/dha--, ZG ZH/dhagenadha--\}$$

where every derivation is explicit. Therefore \mathcal{S}_0 contains a unique substitution that yields the sentence.

If we select $R_0 = \{1,2,3,4,5,8,10,13,14,15,21\}$ we get $\mathcal{S}_{p_{10} \rightarrow q_{10}} = \{\dots, ZD ZE/ZG ZH, \dots\}$, $\mathcal{S}_{p_{15} \rightarrow q_{15}} = \{\dots, ZD/dhin-- , \dots\}$, and $\mathcal{S}_{p_{20} \rightarrow q_{20}} = \{\dots, ZE/dha-- , \dots\}$. Let us call $\mathcal{S}_{15,20} = \mathcal{S}_{p_{15} \rightarrow q_{15}} \cap \mathcal{S}_{p_{20} \rightarrow q_{20}} = \{\dots, ZD/dhin-- , \dots, ZE/dha-- , \dots\}$. For any $s \in \mathcal{S}_{15,20}$, $s(ZD ZE) = s(ZD) s(ZE) = dhin--dha--$. Therefore ZD ZE may never be substituted to ZG ZH, which contradicts $\mathcal{S}_{p_{10} \rightarrow q_{10}}$. Consequently, $\mathcal{S}_0 = \emptyset$ and the derivation is aborted.

Remark: we may also define \mathcal{S}_0 as a maximal (hence *most specific*) consistent conjunctive expression using predicates $(p_i = q_i)$ such that $(p_i \rightarrow q_i) \in R$.

Theorem:

| The class of finite languages coincides exactly with that of RPLs.

Proof:

Each subset of the (finite) set of rules in a RPL yields at most one terminal derivation. Therefore the set of terminal derivations is finite. Conversely, for any finite language there exists a non-embedding right-linear grammar G that generates exactly L . Let $G = (Vt, Vn, S, \mathbf{R})$ where S is the starting symbol and \mathbf{R} a finite set of rules in format $A \rightarrow a$ or $A \rightarrow aB$ such that $A, B \in Vn, A \neq B$, and $a \in Vt$. It is easy to prove that G generates exactly the RPL $L_{\mathbf{R}}(p)$ such that $p = S$, which completes the proof.

Corollary:

The class of RPLs is recursive, and it is closed under union, catenation and intersection.

We can define an effective procedure for constructing $L_{\mathbf{R}}(p) = L_{\mathbf{R}1}(p1) \cup L_{\mathbf{R}2}(p2)$ as follows:

Assume that $L_{\mathbf{R}1}(p1)$ and $L_{\mathbf{R}2}(p2)$ are defined with:

$p1 = S1$ and $\mathbf{R}1 = \{S1 \rightarrow q1, \dots\}$

$p2 = S2$ and $\mathbf{R}2 = \{S2 \rightarrow q2, \dots\}$ in which we renamed variables in $\mathbf{R}2$ so that no variable occurs in both $\mathbf{R}1$ and $\mathbf{R}2$. We construct $L_{\mathbf{R}}(p)$ defined with:

$p = S$ and $\mathbf{R} = \{S \rightarrow S1, S \rightarrow S2\} \cup \mathbf{R}1 \cup \mathbf{R}2$.

The set $L_{\mathbf{R}}(p)$ of terminal derivations of S is the union of the sets of terminal derivations of $S1$ and $S2$, therefore $L_{\mathbf{R}}(p) = L_{\mathbf{R}1}(p1) \cup L_{\mathbf{R}2}(p2)$. In addition, since $\mathbf{R}1$ and $\mathbf{R}2$ contain only well-formed rules, \mathbf{R} contains also well-formed rules. Consequently, $L_{\mathbf{R}}(p)$ is a RPL.

RPLs form a class of formal languages that is closed under union, and there is even a procedure for building a language from its subsets. In addition, the class is recursive and therefore membership tests can be computed. These properties are useful for building representational models that lend themselves to descriptive generalization.

2. Bol Processor grammars

2.1. Pattern rules

Since stroke sequences in North Indian drumming are homophonic, then in any pattern xyz the time predicates $\gamma(x,y)$, $\gamma(y,z)$ and $\delta(x,z)$ are implicit. Therefore the only predicates that need to be made explicit are those denoting repetitions (*equal*) and mirror substitutions (*mirror*).

For example the rewriting rules defining P96 and P192 in §1.4 may be represented with the following pattern grammar:

P96 $\rightarrow X A Y B$ where *mirror*(X,Y) and *length*(X,24)

P192 $\rightarrow C Z1 D A E Z2 D B$ where *equal*(Z2,Z1) and *length*(Z1,12) and *mirror*(C,E)

In this example the meaning of predicates *mirror*, *length* and *equal* is self-explanatory.

It is clear that if two variables linked with an *equal/mirror* predicate appear in the conclusion of a rule, the leftmost variable denotes a sequence of strokes that has been played first and served as a reference. For example, using the following set of rules:

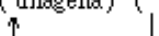
$A \rightarrow B C$ where *equal*(B,C)

$B \rightarrow dhagena$

the terminal derivation *dhagenadhagena* is such that the first *dhagena* has served as a reference for the second. Thus we may notate this sentence:

(= dhagena) (: dhagena)

with the meaning that the parenthesis with an equal sign denotes the reference, and that with a colon is the copy. In fact, in the machine implementation, the content of the second parenthesis is merely a pointer to the reference:

(dhagena) ()


We use the same convention in rules, e.g.:

$A \longrightarrow (= B) (: B)$

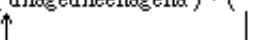
As to notating mirrors, we use an asterisk to indicate that the following sequence between brackets is unvoiced. For example, given the mirror mapping of §1.1, in the following grammar:

$S \longrightarrow (= D) * (: D)$
 $D \longrightarrow dhagedheenagena$

the (only) terminal derivation is

$(= dhagedheenagena) * (: taketeenakena)$

and its internal representation:

(dhagedheenagena) * ()


Using these conventions we can simplify the format of rewriting rules in RPLs:

- (1) If a variable appears several times in the RPL rewriting rule, it shall be represented with repetition markers. For instance, $A \longrightarrow B C B C B$ shall be represented $A \longrightarrow (=B) (=C) (:B) (:C) (:B)$.
- (2) The same variable may appear twice in a rule premise (if not linked with repetition markers), e.g. $A A \longrightarrow q$ is correct but not $(=A) (:A) \longrightarrow q$.
- (3) Mirrors shall be represented with an asterisk.

Using these conventions, the BP grammar generating the RPL language defined under §1.3 is:

- (1) $P192 \longrightarrow (=C) (=Z) (=D) A * (:C) (:Z) (:D) B$
- (2) $A \longrightarrow \text{tagetirakitadhin--dhagena } D$
- (3) $B \longrightarrow \text{tagetirakitadhin--dhagenadhatigegenakadheenedheenagena}$
- (4) $C \longrightarrow \text{dhin--dhagenadha--dhagenadhatigegenakadheenedheenagena } A$
- (5) $D \longrightarrow \text{dhatigegenakateeneteenakena}$
- (6) $Z \longrightarrow \text{tirakitatirakitatirakita}$
- (7) $Z \longrightarrow Z6 \ Z6$
- (8) $Z \longrightarrow Z3 \ Z3 \ Z3 \ Z3$
- (12) $Z3 \ Z3 \longrightarrow \text{dhagenadhin--}$
- (13) $Z3 \ Z3 \longrightarrow \text{dhagenadha--}$
- (14) $Z3 \longrightarrow \text{dhin--}$
- (18) $Z3 \longrightarrow \text{dha--}$
- (22) $Z6 \longrightarrow \text{dheenedheenedheene}$
- (24) $Z6 \longrightarrow \text{dha-dha-dha-}$

in which rules 9, 10, 11, 15, 16, 17, 20, 21 and 23 have been deleted. This representation is more compact although it generates the same language. The syntactic graph of the sentence generated in §1.4 is now:

- (1) If G is a transformational grammar, its *dual* is obtained by swapping the premise and conclusion in every rule.
- (2) If G_1, G_2, \dots, G_n are the subgrammars of the language, a membership test is the result of the *context-sensitive canonic rightmost derivation* of the sentence by G_n, \dots, G_2, G_1 .
- (3) If the membership test has yielded S , the starting symbol, the input sentence is assessed 'correct'. It is 'incorrect' in any other case.

The concept of context-sensitive canonic (leftmost) derivation was defined by Hart (1980:82) for *strictly* context-sensitive grammars, i.e. grammars in which all rule premises contain no more than one variable. This author has extended this definition to all *length-increasing* grammars:

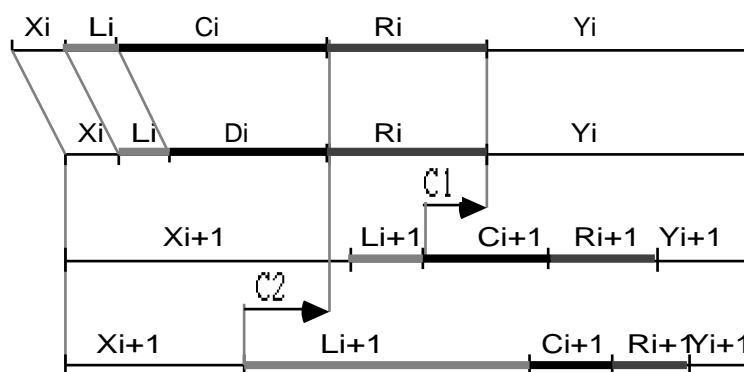
Context-sensitive canonic rightmost derivation: definition

The formal definition of the context-sensitive rightmost definition adapted from Hart (1980) and used in the membership test of LIN grammars is the following:

Let G be a length-increasing grammar. The derivation in G :
 $W_0 \Rightarrow W_1 \Rightarrow \dots \Rightarrow W_n$
 is *context-sensitive rightmost* iff:
 $\forall i \in [0, n-1], W_i = X_i L_i C_i R_i Y_i$
 $W_{i+1} = X_i L_i D_i R_i Y_i$ when applying rule $f_i: L_i C_i R_i \rightarrow L_i D_i R_i$,
 and at least one of the two following conditions is satisfied:
(C1) $|C_{i+1} R_{i+1} Y_{i+1}| > |Y_i|$
(C2) $|L_{i+1} C_{i+1} R_{i+1} Y_{i+1}| > |R_i Y_i|$

In Hart's definition (1980:82), $|C_i| = |C_{i+1}| = 1$, so that C1 may be written $|R_{i+1} Y_{i+1}| \geq |Y_i|$.

The diagram and commentary below illustrate conditions **C1** and **C2**:



Suppose that neither conditions **C1** nor **C2** are satisfied. Since **C2** is not true, rule f_{i+1} could have been applied before f_i as $L_{i+1} C_{i+1} R_{i+1}$ would be a substring of $R_i Y_i$. Besides, since **C1** is not true, applying rule f_{i+1} would only modify Y_i without changing the context R_i . In such a case the order of application of f_i and f_{i+1} might have been inverted. This change in the order would have been justified since all symbols rewritten by f_{i+1} are to the right of those rewritten by f_i .

Now, an explanation of how the inference engine of the BP handles ambiguity in bottom-up parsing will be given. Suppose that for a working string W_i there are two candidate rules:

$$\begin{array}{lll}
 \mathbf{f}_i & L_i C_i R_i & \rightarrow L_i D_i R_i \\
 \mathbf{f}'_i & L'_i C'_i R'_i & \rightarrow L'_i D'_i R'_i
 \end{array}
 \quad \text{where } X_i L_i C_i R_i Y_i = X'_i L'_i C'_i R'_i Y'_i = W_i$$

The selection criterion is the following: f_i will have priority over f'_i if one of the following conditions (in this order) is satisfied:

- (D1) $|X_i L_i C_i| > |X'_i L'_i C'_i|$
- (D2) $|X_i L_i C_i R_i| > |X'_i L'_i C'_i R'_i|$
- (D3) $|L_i C_i R_i| > |L'_i C'_i R'_i|$
- (D4) $i > i'$

It can be proved (Bel 1987a:8) that once D1 has been considered, D2 is no longer relevant and ambiguity may therefore be handled by D3 and D4. D3 makes a decision on the basis of the length of the conclusions of the two rules, and D4 is a final arbitrary decision that takes into account the order in which the rules appear in the grammar.

To augment efficiency, D3 is not considered by the inference engine of the BP; it is therefore the task of the analyst to take into account the following partial ordering of rules:

‘Chunk’ rule:

| In a BP grammar, the conclusion of rule f_i may not be a substring of the conclusion of f_j such that $j < i$.

There are a few more restrictions on rule formats imposed by the membership test procedure (Bel 1987a-b). One is that *right contexts may only contain symbols of an alphabet external to the grammar*.

2.3. Templates

Sentences can only be parsed if they have been entered into the editor complete with structural information, i.e. brackets, asterisks, etc. Yet in view of the fact that the Bol Processor has to perform membership tests on large amounts of data comprising examples to which it is not always easy to assign a structure, this limitation is held to be unacceptable. In response to this, the inference engine of the Bol Processor is able to generate templates from a grammar, i.e. a list of possible structures in which each dot represents a terminal symbol of one unit. Templates are enumerated and stored in the grammar file, for instance:

- [1] (=.....)(=.....)(=.....)*(.....)(:.....)(=.....)
- [2] (=.....)(=.....)*(.....)(:.....)
- [3] (=.....)(=.....)*(.....)(=.....)

In analysis the Bol Processor takes a new sentence and superimposes it on each template in strict order. A membership test is performed each time the sentence matches a template, so allowing for any structural ambiguity to be assessed. An example of template matching may be found in Kippen & Bel (1989c: appendix).

2.4. Stochastic model

The only realistic method for testing a grammar with an expert musician is to instruct the machine to produce one randomly chosen sentence at a time. If the sentence is assessed correct, the procedure is invoked again and another sentence is generated. Generally, the grammar is considered to be satisfactory if all sentences generated within a few sessions have been accepted by the expert.

Since the correctness of a grammar can never be fully assessed — indeed, like musicians themselves, machines may be allowed casual mistakes — it is important to enable the stochastic

production process to generate sentences from a wide and representative subset of the language. This can be achieved by weighting the decisions of the inference engine.

Formal grammars offer the possibility of defining consistent probabilistic models that apply to a very wide class of languages. We developed such a model as a response to the need to inhibit rules in context-free grammars. This model is derived from probabilistic grammars/automata as defined by Booth & Thompson (1973), the difference being that a *weight* — within the range [0,255] — rather than a *probability* is attached to every context-free rule. The rule probability is computed from weights as follows: if the weight is zero then the probability is zero; if the weight is positive then the inference engine calculates the sum of weights of all *candidate* rules, and the rule probability is the ratio of its own weight to the sum. Candidate rules are those whose premise is a substring of the work string. Consider, for example, the set of rules

[1]	<100>	V3	—>	dhagena
[2]	<100>	V3	—>	dhatrkt
[3]	<50>	V3	—>	dha--
[4]	<5>	V3	—>	dhati-

in which the sum of the weights is $100+100+50+5 = 255$. The probability of choosing rule [4] in the derivation of a string containing *V3* is therefore $5/255 = 0.0196$. Using weights instead of probabilities has the advantage that it does not presuppose the sum of coefficients of all candidate rules to be 1.

Weights (and their associated probabilities) are used in *modus ponens* to direct the Bol Processor's production along paths more likely to be followed by musicians. In some context-free grammars — those that fulfil the *consistency* condition expressed by Booth & Thompson (1973:442) — they may be used for computing a *probabilistic sentence function*, i.e. a coefficient representing the likelihood of occurrence of each sentence in the language. Grammars that are constructed in a systematic way (or generated from sample sentences, see Kippen & Bel 1989b:205) are good examples of consistent probabilistic grammars.

Another remarkable feature of consistent grammars is that rule probabilities can be inferred from a set of sentences (Maryanski & Booth 1977:525). Given a correct grammar and a subset of the language that this grammar generates (for instance a sample sequence taken from a performance of an expert musician), rule weights are inferred as follows: let all weights be reset to zero; then analyse every sentence and increment by one unit the weights of all rules used in the derivation. (The algorithm described here is more general than the one devised by Maryanski and Booth, since the latter requires the choice of a sample set in which *all* rules have been used.)

Evidently, rules that are never used in the analysis of a sample set remain with weight (and therefore probability) zero, which inhibits their use in *modus ponens*. Those rules may be scrutinized to see whether they are incorrect or whether they point to fragments of the language that have not yet been explored. To test this, their weights are set to a high value so that the Bol Processor is forced to generate sentences that either have never been assessed or at least are not being considered by the informant at the time.

An example of weight inference based on the *qa'ida* introduced in §1.2 may be found in Kippen & Bel (1989a).

Using weighted rules resulted in a marked improvement in the quality of the generated music. This went a long way towards solving the problem of musical credibility encountered in earlier experiments, a problem that arose from the complete randomness of the generative process.

2.5. Recent developments of the Bol Processor

These developments have taken place as the result of collaboration with Western composers, more specifically musicians working with digital synthesizers. Many have felt a strong need to implement general abstract models of musical structures. A new version of the Bol Processor has been implemented in C language on the Macintosh. 'BP2' accepts polyphonic structures in a format loosely related to Diener's *T-trees* (1988). Unlike T-trees, however, where each individual terminal (e.g. a note or a stroke) is viewed as an object (in an *object-oriented* programming environment), BP2 remains based on strings. Structural units of overlapping events may be represented as sets or trees. Several λ -free homomorphisms may be defined on the same alphabet. Wildcards (local variables that may be instantiated on $V_n \cup V_t$) can be used in the left argument of context-sensitive rules to represent patterns. BP2 is also able to handle *metagrammars*, i.e. grammars that generate grammars.

A MIDI interface is under study so that the sentences generated may be immediately checked in the way they actually sound.

3. Conclusion

Pattern grammars (which may be imbedded in production rules, logic descriptions, tree algebra, etc.) are not only powerful tools for music analysis. They also lend themselves to descriptions of creative/evaluation processes. In the phenomenological approach, a pattern is viewed more as a *dynamic* than static relation:

(La forme) n'est pas la somme des détails intégrés dans l'ensemble qui constitue l'oeuvre. Elle appartient au niveau des structures, c'est à dire des principes. Elle s'identifie avec le schème d'organisation qui suggère l'assemblage des parties en considération de leurs liaisons avec les lois propres du schème organisateur. La forme est liée aux principes de cohésion du système, à la réalisation d'un dynamisme particulier.

(Vecchione 1984:11-12)

Descriptions may be *normative* (i.e. based on some kind of rigorous reasoning) when dealing with highly constrained systems (e.g. improvisation in traditional music) or *empirical* (i.e. based on multi-criteria classification) when applied to the modelling of a compositional process.

The problem lies less in finding a 'universal' abstract representation of music, than in delineating forms of musical 'thinking' that may be operative in the design of tools for computer-aided music creation and performance.

Acknowledgements

I am indebted to Prof. Bernard Vecchione and Dr. Jim Kippen for their help in providing musical examples and the *raison d'être* of this paper. Their competence in music theory, practice, and in the history of musical analysis has been essential for the difficult task of linking formal models with musical concepts.

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