A MATHEMATICAL MODEL OF THE SHRUTI-SWARA-GRAMA-MURCCHANA-JATI SYSTEM

E. James Arnold

1. Summary

The first section of this paper considers the mathematical and musical principles by which the *shruti* positions (note positions in the system of "just intonation") are determined. A system of shorthand tuning symbols is given by which the tuning procedures for each of the *shruti* positions is concisely expressed. Using these symbols to summarize the appropriate tuning procedures, the positions of the *shrutis* are given in tables of families.

Then follows a description of a simple mathematical model of the *shruti-swara-grama-murcchana* system of Bharata, Dattila, and Sharangadeva. A study of the harmonic structure of the *gramas* (fundamental scales) reveals that the principle scale-types employed in Hindustani classical music form a 'main-sequence' of scales directly relatable to the harmonic structure of the ancient *gramas*. This is summarized in a table of the 'extended *murcchana* series' of scales derivable from the Ma and Sa *gramas*.

2. Introduction

The structure of the natural relations of consonance existing in musical sound is rigorously determinable from simple axioms. Because of the transitory nature of sound have been it is difficult to grasp these relationships conceptually. Hence, in spite of the subjects utter simplicity, the question of *shrutis* and scales remains for most musicians a mystery despite the enormous practical value which this knowledge has. Alain Danielou has put the case for just intonation excellently:

> Even if we leave aside the role of music as a means of spiritual realization, the effect of musical chords and modes (ragas) is much more far reaching than our ears are able, at first, to allow us to perceive.

> Our ear can apparently be satisfied by a very approximate accuracy. But a perfectly accurate interval does not react only on our ears, it produces a transformation in all the cells of our body...If, we utilize habitually inaccurate intervals, on the ground that our ear does not clearly perceive the difference, the effect that those sounds will produce on our organism can well be the opposite of that which our complacent ears persist in accepting. It is with our mind only that we cognate this inaccurate music which leaves us tired, contracted by the unconsious effort of adaptation instead of being agreeably transformed by the beneficial influence of harmony. This is why to disregard small differences in intervals has very grave consequences with regard to the deeper effect of music. (Danielou 1943: 15).

3. Principles for the divison of the octave

The '*shrutis*' as they are called, are nothing more than potential note positions.[1] Where a given note (*swara*) should be, pitchwise, is, from a geometrical point of view, a rigorous function of simple laws of consonance (*samvad*).

The pitch relations available within the set of tones defined by a fundamental tone (Sa or Do) and its first five harmonics (where the first harmonic is identical to the fundamental) constitute the basic elements, which can be systematically manipulated to define all of the *shrutis* of the musical gamut. These pitch relations define the following key intervals:

1. Let Sa (Do) equal 'n' vibrations per second, then, n/n defines the space of the unison (SA);
2. 2n/n defines the space of the octave (Sa-Sa);
3. 3n/2n defines the space of the perfect fifth (*pancam*),
4. 4n/3n defines the space of the perfect fourth (*madhyam*),
5. 5n/4n defines the space of the harmonic major third (*swayambhu gandhar*).

The perfect fourth is an 'inversion' of the perfect fifth. That means that if one starts from the octave (Sa or Do_2) and *descends by a perfect fifth,* he will arrive at the note a perfect fourth above the base. By using this principle, and taking the inversion of the harmonic major third we can add the last necessary key interval to the set:

6. **8n/5n defines the space of the harmonic minor sixth (*komal dhaivat*)**

From this axiomatic set of intervals the rest of the notes and their *shrutis* can be defined.

[1] cf. Lath 1979: 207-209

4. The tuning symbols

To express in short-hand the exact tuning procedure to follow to tune (or define mathematically) the rest of the *shrutis,* we have used a system of tuning symbols for which we are indebted to Jacques Dudon.

The system of symbols should be read as follows:

1. Read $\overset{\cdot\cdot}{n}$ as "the 'nth' ascending fifth from Sa (Do)."
2. Read $\underset{\cdot}{n}$ as "the 'nth' descending fifth from Sa (Do)."
3. Read $\frown$ as "a major third (suddha Ga) above Sa (Do)."
4. Read $\smile$ as "a major third below Sa (Do)." (equal to komal dhaivat)"
5. Read $\overset{\frown}{n}$ or $\underset{\frown}{n}$ as "the 'nth' ascending, or descending, fifth from the major third above Sa (Do)."
6. Read $\overset{\smile}{n}$ or $\underset{\smile}{n}$ as "the 'nth' ascending, or descending, fifth from the major third below Sa (Do)."

Several examples will make this system clear. The symbol should be read as "the fifth successive ascending fifth from Sa. That means that starting from Sa, one should first tune a perfect fifth. That is fifth number 1. From that note in turn, one tunes another fifth, making fifth number 2. From that note in turn, one tunes again a fifth making fifth number 3, and so on until fifth number 5 is reached. One can procede in similar fashion but in the downward direction in the case of descending fifths. Note well that a fifth up is equal to a fourth down; and *vice versa,* a fifth down is equal to a fourth up. Instead of taking a series of fifths up (or down) which rapidly moves one into octaves far from the starting pitch, one can alternate between fifths up and fourths down to stay in the original octave, while, in effect following a chain of fifths up. And similarly, one can alternate between fifths down and fourths up to stay in the original octave, while following a chain of fifths down.

Take for example, the symbol $\underset{\frown}{2}$. It should be read as "the second descending fifth from the major third." Tune it as follows: first tune the major third (suddha GA) from Sa. Then tune a fifth down from that, making the first fifth down, and then tune a fourth up from that making, in effect, the second fifth down. One will have reached Re^3 (Re 10/9).

Take for another example, the symbol $\overset{\smile}{2}$. It should be read as "the second ascending fifth from the major third below Sa." Tune it as follows: first tune a major third down from Sa. Then tune a fifth up from that, making the first fifth up, and then tune a fourth down from that making, in effect, the second fifth up. Here one will have reached Ni^3 (Si 9/5).

Take one more example. Read the symbol $\overset{\frown}{\hat{2}}$ as "the second ascending fifth from the major third," and tune it as follows: first tune the major third above Sa. From this tune one fifth up, making the first fifth up, and from that tune a fourth down making, in effect, the second fifth up. Here one will reach Ma^3 (the augmented fourth 45/32).

5. The determination of the *shrutis*, general division of the *gamme*

Using this short-hand notation system, four procedural series, by which the *shruti* positions of the twelve notes are determined, are summarized in the diagram of the harmonic logic and the tables of *'shruti* families' which follow. Two of these series we call 'Pythagorean families' because they are based on the cycles of fifths and fourths respectively. The two series give *shrutis* in what we call 'harmonic families' because they take their start from the harmonic major third either above or below Sa. The tables summarize for each *shruti* position, its tuning procedure, its frequency ratio, period ratio to Sa, its value in terms of Savarts, cents, degrees, and 'percents'. Percents are the expression of the value of the note's ratio in base 2 logarithms multiplied by 100. They indicate the position in terms of a percentage of the octave space (cf. Arnold 1974).

I. Pythagorean Major Shruti Positions

Swara	Note	Tuning	Ratio	Period	Savarts	Cents	Degrees	Percents
Sa	Do	--	1/1	1.000	0	0	0	0
Pa^4	Sol	$\hat{1}$	3/2	1.333	176.09	702	210.59	58.50
Re^4	Re	$\hat{2}$	9/8	1.778	51.15	204	61.17	16.99
Dha^4	La^+	$\hat{3}$	$\frac{27}{16}$	1.185	227.24	906	271.76	75.49
Ga^4	Mi^+	$\hat{4}$	$\frac{81}{64}$	1.580	102.30	408	122.35	33.98
Ni^4	Si^+	$\hat{5}$	$\frac{243}{128}$	1.054	278.40	1110	332.93	92.48
Ma^4	$Fa\S^+$	$\hat{6}$	$\frac{729}{512}$	1.405	153.47	612	183.52	50.98

Key: § = sharp
+ indicates one comma higher than Zarlino's position
b = flat
- indicates one comma lower than Zarlino's position.

II. Pythagorean Minor Shruti Positions

Swara	Note	Tuning	Ratio	Period	Savarts	Cents	Degrees	Percents
Sa	Do	--	2/1	1.000	301.03	1200	360.00	100.00
Ma^1	Fa	$\check{1}$	4/3	1.500	124.94	498	149.41	41.50
Ni^1	Si b$^-$	$\check{2}$	$\frac{16}{9}$	1.125	249.87	996	298.83	83.01
Ga^1	Mi b$^-$	$\check{3}$	$\frac{32}{27}$	1.687	73.79	294	88.24	24.51
Dha^1	La b$^-$	$\check{4}$	$\frac{128}{81}$	1.266	198.72	792	237.65	66.02
Re^1	Re b$^-$	$\check{5}$	$\frac{256}{243}$	1.898	22.63	90	27.07	7.52
(Pa^1 = Ma^3	Sol b$^-$	$\check{6}$	$\frac{1024}{729}$	1.426	147.57	588	176.48	49.02)
	Fa § --see below--							

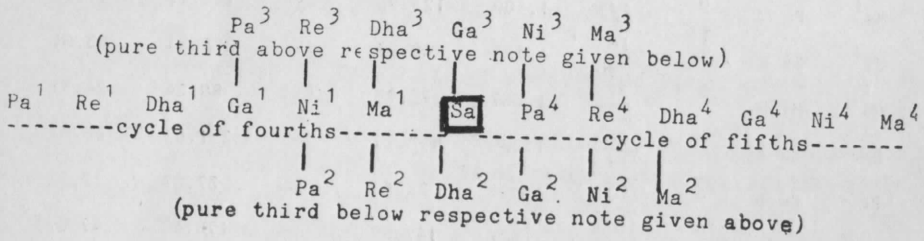
III. Harmonic Major Shruti Positions

Swara	Note	Tuning	Ratio	Period	Savarts	Cents	Degrees	Percents
Sa	Do	--	1/1	1.000	0	0	0	0
Ga^3	Mi	$\frown$	5/4	1.600	96.91	386	115.89	32.19
Ni^3	Si	$\overset{\frown}{\hat{1}}$	$\frac{15}{8}$	1.067	273.00	1088	326.48	90.69
Ma^3	Fa §	$\overset{\frown}{\hat{2}}$	$\frac{45}{32}$	1.422	148.06	590	177.07	49.19
Dha^3	La	$\overset{\frown}{\check{1}}$	5/3	1.200	221.85	884	265.31	73.70
Re^3	Re^-	$\overset{\frown}{\check{2}}$	$\frac{10}{9}$	1.800	45.76	182	54.72	15.20
Pa^3	Sol^-	$\overset{\frown}{\check{3}}$	$\frac{40}{27}$	1.350	170.69	680	204.13	56.70

IV. Harmonic Minor Shruti Positions

Swara	Note	Tuning	Ratio	Period	Savarts	Cents	Degrees	Percents
Sa	Do	--	2/1	1.000	301.03	1200	360.00	100.00
Dha^2	La b	$\smile$	8/5	1.250	204.12	814	244.11	67.81
Ga^2	Mi b	$\overset{\smile}{\hat{1}}$	6/5	1.667	79.18	316	94.68	26.30
Ni^2	Si b	$\overset{\smile}{\hat{2}}$	9/5	1.111	255.27	1018	305.28	84.80
Ma^2	Fa^+	$\overset{\smile}{\hat{3}}$	$\frac{27}{20}$	1.481	130.33	520	155.88	43.30
Re^2	Re b	$\overset{\smile}{\check{1}}$	$\frac{16}{15}$	1.875	28.03	112	33.52	9.32
Pa^2 = Ma^4	Sol b	$\overset{\smile}{\check{2}}$	$\frac{64}{45}$	1.406	152.97	610	182.93	50.81
	$Fa\S^+$ --See above--							

HARMONIC LOGIC OF THE SHRUTI POSITIONS



The above tables summarize the positions of the *shrutis,* according to their natural families. A few remarks are in order.

Note that two equivalences are stated. The *swaras* Pa^1 and Pa^2 are said to be equivalent to Ma^3 and Ma^4 respectively. The Indian tradition does not reckon the existence of the diminished fifth (*komal pancam*) even though it is the natural inversion of the augmented fourth (*tivra madhyam*) which is recognized. We have included it in the list, nevertheless, because at this point lies a small error, the schisma, which is equal to one-tenth of a comma (2 cents). It is an error so small that it is virtually inaudible, and practically speaking can be ignored. Doing so, one can point out two connections between the four *shruti* families: namely that "Pythagorean major *shrutis*" and "Harmonic minor *shrutis*" both belong to the cycle of fifths; and Pythagorean minor *shrutis* and Harmonic major *shrutis* both belong to the cycle of fourths (ignoring the difference of the schisma). It is useful to make this equation for the sake of scale systematics, a subject which follows.

Note too, that considering Sa as one position, there are 22 additional positions indicated making what appears to be a total of 23 *shrutis,* where the tradition recognizes only 22. This difference is accounted for by the fact that the value of 1 *shruti* is ambiguous. Assuming that a 4-*shruti* interval equals a major whole tone (9/8), a 3-*shruti* interval equals a minor whole tone (10/9), and a 2-*shruti* interval equals a semitone (16/15), one *shruti,* can mean either a comma (81/80), a limma (256/243), or a minor semitone (25/24), depending on its position. However since a comma plus a minor semitone is equal to a limma, the confusion arises. Between Ma^1 and Pa^4, the space of a major whole tone (4 *shrutis*), there appear to be *five shrutis* when in reality there are only four. The term '*shruti*' anyway is ambigous and sometimes means an interval, and sometimes means a note-position (cf. Lath 1979 : 203-206).

PART 2: THE DESCRIPTION OF THE MODEL

This device is a logarithmic model of the exact relationships pertaining to notes and scales in the system of 'just' or natural intonation, in the terms expressed by the ancient musicologists of India, Bharata, Dattila, and Sharangadeva. Its construction is based on the idea of mounting an inner movable wheel marked with the note positions of a basic scale (*grama*) against a fixed background wheel marking the natural division of the octave into *shrutis.*

To put the model together for use as a simple scale-calculator cut-out the *grama* wheel mount both it and the *shruti*-wheel by means of a suitable stiff backing and fasten the *grama* wheel to the *shruti* wheel, by means of a small rivet or a bent piece of a paper-clip, in such a way that the *grama* wheel is movable.

The model displays at a glance the various logical and consistent relationships possible among justly intoned musical scales. A slight error is sometimes introduced by the schisma, but practically speaking, it can be ignored. By positioning any *swara* of the *grama* wheel to match with the Sa of the background *shruti* wheel you can read out the *shruti* positions of the notes of the *murcchana* (mode) which starts from the note chosen. The values of the intervals of the *murcchana* are directly translated into terms of Sa, making it easy to see what scale-type the *murcchana* represents, and what are the *shruti* positions of its notes.

1. Description: The fixed wheel

The outer, or fixed circle indicates the division of the octave in terms of the *shrutis.* It is marked with the letters 'L' for limma (256/243), 'M' for minor semitone (25/24), and 'C' for comma (81/80. These intervals indicate the tonal distance between the points of manifestation (*sphota*) of a note. The *sphota* points are named by the designations 'Re^1, Re^2, Re^3, Re^4, etc. (following the *shruti* tables on pages 33 and 34). *Swara* positions with the numbers 1 and 2 are *komal* (minor) and *swara* positions with the numbers 3 and 4 are *tivra* (major). Each position is further identified by its mathematical ratio inscribed exterior to the outer ring, by its period ratio, and by the set of tuning symbols which describe how to tune the position.

2. Description: the movable *grama* wheel

The movable *grama* wheel contains the following information. Its first, or outer ring divides the octave chromatically into twelve notes, using a particular basic chromatic scale. The logic for selecting this scale is this: given twenty-two *shruti* positions for eleven of the *swaras* (the base, Sa, being fixed), there are 2^{11} or 2048 possible chromatic scales. Most of these, however,

lack good consonant relations among their notes. By applying the principle of maximum internal consonance, all but twelve can be eliminated. The chromatic scale which is marked on the outer ring of the *grama* wheel is that sole member of the remaining set of twelve into which the *Ma-grama* of the ancient system of musical acoustics will fit. For this reason, we refer to it as the '*Ma-grama* chromatic scale'. It is that chromatic scale achieved by taking one complete round of the cycle of fourths (having all of its notes in Sa-Ma *bhava*). Note that the Pa^4 of the *Sa-grama*, the *cyuta* Sa, and the *cyuta* Ma are extraneous to this scale, but are added for convenience.

The lines in the second circle, moving inwards, mark the division of the octave diatonically according to the Sa and Ma *gramas*. The *suddha swaras* (natural notes) are marked in solid lines, and the *vikrit swaras* (altered notes) are marked in dotted lines. This space also contains the letter names of the notes: Sa, Re, Ga, Ma, Pa, Dha, Ni (Do, Re, Mi, Fa, Sol, La, Si).

In the third ring inwards of the movable wheel the numbers of *shrutis* contained in the *swaras* of the ancient *gramas* are indicated (cf. *Dattilam* 11A-15B, Lath 1978: 218); in the Sa *grama* the order is 4-3-2-4-4-3-2, while in the Ma-*grama* it is 4-3-2-4-3-4-2, for the *swaras* Sa-Re-Ga-Ma-Pa-Dha-Ni.

3. The Inner harmony of the *gramas*.

The innermost circle of the *grama* wheel indicates the underlying harmonic structure of the two basic *gramas*. Both Ma and Sa *gramas* are composed of two consonant chains of perfect fifths and fourths which are offset from each other by a perfect major third. Note that the characteristic of the *Sa-grama* is that its Pa (Sol) belongs to the chain of consonant notes which includes Sa (Do), while in the *Ma-grama* the Pa belongs to the *other* chain (and is consonant with Re^3). To be complete, the scale of the *gramas* must have a Pa, but it cannot be decided, *a priori*, that Pa must belong to one chain or the other. Logically, both alternatives are equally valid. It is this option, which only Pa has in this scale, which poses the crucial question.

If Pa belongs to the Sa-chain, then the notes of the lower tetrachord, i.e., the lower four notes, Sa Re^3 Ga^1 Ma^1, have a parallel muiual consonance to the notes, Pa^4 Dha^3 Ni^1 Sa, of the upper tetrachord. This scale has what are called 'disjunct' or separated tetrachords at Ma-Pa.

If, on the other hand, Pa belongs to the Re-chain, then the notes of the lower tetrachord, Re^3 Ga^1 Ma^1 Pa^3 are paralleled by the notes Dha^3 Ni^1 Sa Re^3 of the upper tetrachord. This scale, then, has disjunct tertachord at Pa-Dha.

Depending on the *shruti*-positions of its notes, a given scale-type may show more than one set of harmonically parallel tetrachords (*angas*). These different possibilities are exploited melodically by different ragas which use the same scale-type, but which explore it differently by their melodies. The genius of the ancient Indian analysis of musical scales lies in the fact that this central question posed by the logic of the equal validity of the two positions for the *grama pancam* (Pa) is repeated in its poignancy on the crucial note of all of the scales in the main sequence of the diatonic and related *grama* scale-types in which, by far, the bulk of the world's music is played.

4. Validity of the model from the point of view of *Sangita Shastra*.

That the system of musical scale relationships represented on this model is an accurate reprsentation the system of relationships descirbed in the ancient *grama-murcchana system* of early *Sangita Shastra* is argued on the following two criteria: (1) the system of note positions dividing the gamut into *shrutis* is acoustically sound. (2) If the instructions of the *shastras* are followed placing the notes of Ma or Sa-*grama* on their proper *shruti* positions, the complete set of *murcchanas* require these, and only these, positions to express their notes at will their proper *shruti*-positions.

5. *Grama-related scale-types* extension of the *grama-murcchana* system to rationalize the commo l mod s of Hindustani music:

The principle scale-types (*thaats*) which are commonly recognized in Hindustani classical music belong within the logic of the *grama-murcchana* system. *Murcchana* is the systematic transformation of a *grama* scale-type into other scale-types by tonic shift (starting scales from different notes of the *grama*, eg. Sa to Sa, Re to Re, Ga to Ga etc.). The diatonic *thaats* of Hindustani music, *kalyan*, *bilaval*, *khamaj*, *kaphi*, *asavari*, and *bhairavi* (equivalent to F, C, G, D, A, and E modes respectively) are identical to the Ga, Ni, Ma, Sa, Pa, and Re *murcchanas*. The 'tritonic' scale- types (so I call them because they contain an additional tritone) *marva*, *purvi*, *todi*, *lalit*, and even *bhairao* with some extra qualifications, belong to a first-order extension of the paramenters of the *grama murcchana* system.

To account for these scale-types in terms of the *gramas*, it is necessary to begin *murcchanas* from the *vikrit swaras*, something which was not allowed in the ancient system. In addition, it is necessary to add other *vikrit swaras* to the *grama* which were not originally mentioned. Lastly, it is necessary to drop the *suddha swaras* from the *gramas* which have been replaced by *vikrits*.[1] It is not difficult to determine the positions of the extra *vikrit swaras* since the *Ma-grama* is demonstrably a transilient (lesser note version) scale

[1]Note too, that the rule of *avanashi madhyama* is occasionally broken.

to the '*Ma-grama* chromatic scale, and the *Sa-grama* can be defined from the *Ma-grama* by *murcchana* shift.

The following table of the 'main sequence' scales belonging to the *grama-murcchana* system, thus extended, shows that the principle *modes* of North Indian music form a logical sequence of scale-types whose inner harmony is directly relatable to the *gramas*. For the *murcchanas* starting from each note of the chromatic *Ma-grama*, the table indicates which *mode* is produced, from the *suddha grama*, from the *vikrit grama* taking *antara* Ga (Ga^3), and from the *vikrita grama* taking both *antara* Ga and *kakali* Ni (Ni^3) The table also indicates what modifications to the *grama* are necessary, when they are so.

SCALES OF THE EXTENDED GRAMA-MURCCHANA SERIES

step	Ma-grama chromatic murcchana starting from:	(1) śuddha grama	(2) vikrit grama (antara)	(3) vikrit grama (antara kakali)	adding to grama	dropping from grama
1.	Sa	kaphi	khamaj	bilaval	--	--
2.	Ma^1	khamaj	bilaval	kalyan	--	--
3.	Ni^1	bilaval	kalyan	marva	3: N^1	S
4.	Ga^1	kalyan	marva	purvi	2: G^1_1 3: G^1_1 N^1	2: M^1_1 3: M^1_1 S
5.	Dha^1	marva	purvi	todi	1: D^1_1 2: D^1_1 G^1_1 3: D^1 G^1	1: N^1_1 2: N^1 M^1_1 3: S M^1
6.	Re^1	purvi	todi	--	1: R^1_1 D^1_1 2: R^1 D^1	1: G^1_1 N^1_1 2: G^1 N^1
7.	Ma^3	todi	--	a. lalit b. bhairao	1: M^3_3 R^1 3a: M^3_3 3b: M^3 R^1	1: N^1_3 G^1 3a: D^3_3 G^1 3b: D^3 S
8.	Ni^3	--	--	a. lalit b. bhairao c. bhairavi	3a: G^1_1 N^1_3 3b: G^1_1 M^3 3c: M^3_3	3a: R^3_3 D^3_3 3b: R^3_1 M^1 D^3 3c: M^1
9.	Ga^3	--	komal pancam§	a. todi b. bhairavi	3a: -- 3b: --	3a: D^3 R^3 3b: --
10.	Dha^3	komal p.	bhairavi	asavari	--	--
11.	Re^3	bhairavi	asavari	kaphi	--	--
12.	Pa^3	asavari	kaphi	khamaj	--	--

We call this scale *'komal pancam'* (equivalent to B-mode) because its distinguishing characteristic is t at its Pa (Sol) has shifted to the diminished, or *komal* position. There is no name for this scale in the Hindustani music tradition, nor are there any ragas in this scale-type.

6. Using the model.

This model has several practical uses.

It can be used to determine the nature of an interval between any two *shruti* positions, much faster and more simply than by calculation. It can be used to study one raga's melodic movements in terms of another raga's movements (by *murcchana* shift) It can help to understand harmonic relationships of the scale degrees in ragas mentioned in *Sangitashastra*. Perhaps its most interesting use is to help see harmonic parallels between the two *angas* (tetrachords) of a ragas scale. Insight into the meaning of this phenomenon helps for studying possible raga harmonic consonances (*samvad* among respective note positions in the two *angas*) in the search for aesthetically rational new ragas possibilities.

7. A word of caution

The work herein is a geometry of musical scales in just intonation created by applying insights of the ancient Indian musicological tradition to an unambiguous set of terminals (the note frequency ratios) defined by the Greek musicologists. The real phenomenon of intonation in Hindustani Classical Music as practised is much more amorphous and untidy than any geometry of course, as recent empirical studies by Levy (1982), and Arnold and Bel (1983) show.

In reality there are no steady tones in Hindustani music : the actual tone is in a state of constant adjustment at every moment. The distinguishing characteristic of any note of raga music is not a frequency ratio, but a movement as an evolving sound in time over a context defined range of tonal space.

Therefore, the proper use of musical scale geometry is to help understand the linguistic and aesthetic implications of different accoustical divisions of the tonal continuum. Another way to put is that the geometry of musical scales is a tool for mapping and understanding the tonal terrain according to linguistically and aesthetically significant landmarks. Frequency ratios are primarily *names*, not precise measurable positions.

8. The harmonic characteristics of the *jatis*.

Once the set of acoustic relationships of the *grama-murcchana* system is known, it is not a difficult matter to understand the harmonic characteristics of the *jatis*, ancient "melody types" often thought to be the precursors of ragas. See Shringi and Sharma, *Sangita Ratnakara of Sarngadeva* pp. (266-368.) Only it is necessary to accept that the *nyasa swara* (note of tension release) of the *jati* defines the *murcchana* from which the *jati* is born. Since,

in fact, the names of the *shuddha jatis* are tied to their *nyasa swaras,* this too suggests that they should be tied to the *murcchanas* beginning from those *nyasa swaras.*

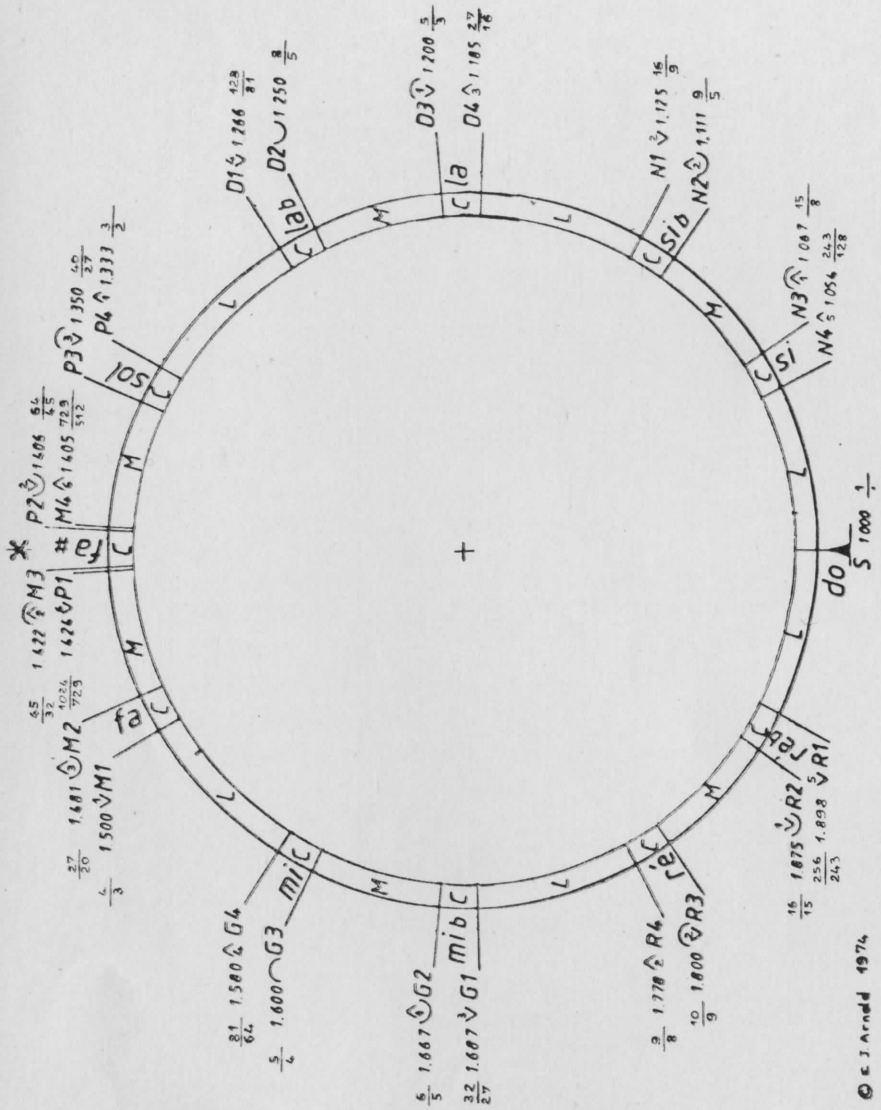
Then too, it must be born in mind that the *lakshanas* (descriptions) of the *jatis,* which tell what *swaras* are important, which *swaras* can be omitted, starting notes for melodies, etc. in the *jati* are told in *referrence to the grama from which the jati is born.* Strictly speaking the *gramas* belong to that aspect of *nada* (vibration) which is *anahata* ("unstruck"). That means to say that the "*grama*" can never be heard *as a musical scale.* What can be heard as a musical scale is not the *grama,* but any of its *murcchanas.* The *grama* scale, as it can be heard, therefore is the *sadja murcchana* scale of the *Ma or Sa-grama.* In the case of *sadji jati* (born of Sa-*murcchana*) the *lakshanas* have a direct meaning and there is not the problem of distinguishing between the *lakshana* as it applies to the *grama* and the *lakshana* as it applies to the resulting scale-type obtained by the *murcchana* chosen. In the case of the other *jatis,* one must not forget to "translate" the *lakshanas* to follow the *murcchana* shift. It is easy to err on this point. Lath, for example has said, "in *gandharva, sadja* was not the all-important basic note as it is today...[since] *sadja* could be omitted..." (Lath 1978:249). *Sadja,* in the sense of a tonic, can never be omitted, and it is likely that *sadja* has always had the function of a tonic, as it has today. When a musician sang *gandhari jati,* and when he arrived at the note *gandhar* of the Ma-*grama* in which he was singing, which is the *nyasa swara* of the *jati,* no doubt he called that pitch "Sa". Musicians who had the insight into the structure of musical scales as deep as is evidenced in the *grama-murcchana* system would hardly have had to change the pitch of their natural tonic, changing *murcchanas* as one might on a harp, in order to sing a different mode (*jati*)! It is clear that the purpose of the *shruti-swara gramamurcchana jati* system is show hat the harmonic relations in all (or at least most) of the useful musical scales can be stated in terms of one basic pair of tunings for a single scale-type. In this sense this system should be understood as an attempt to formulate a "deep grammar" of the science of musical scales. For this it should be regarded as a conceptual achievement on a par with what has been done in the science of linguistics by the Sanskrit grammarians.

REFERENCES

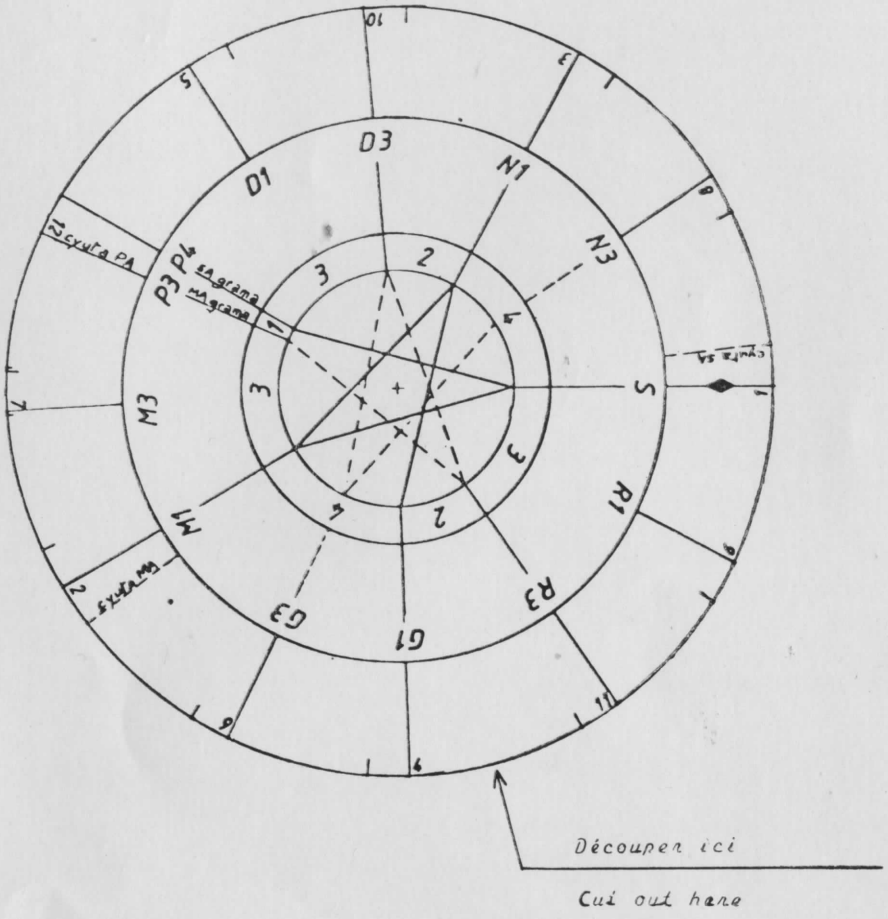
1. Arnold, E.J. "Notes on the construction of a simple shruti-swara-grama-yantra". *AIIS Quarterly Newsletter*' New Delhi : American Institute of Indian Studies, 1974.
2. Arnold, E.J. and Bel, B. "A scientific study of North Indian Music". *NCPA Quarterly Journal,* vol. XII (1983) Nos. 2 & 3, Bombay : The National Centre For the Performing Arts, 1983.
3. Danielou, A. *Introduction to the study of Musical Scales.* London : the ASIA Society, 1943.
4. Lath, M. *A Study of Dattilam.* New Delhi : Impex India, 1978.
5. Levy, M. *Intonation in North Indian Music.* New Delhi : Biblia Impex, 1982.

HARMONIC CHARACTERISTICS OF THE JATIS

jati name	murchana	grama	shadav audav	amsha	modern scale name	resulting scale	shadav audav	amsha: (vadi)
	reference to grama		omitting			reference to resulting scale	omitting	
Shadji	Sa	Sa	--	S G M P D	Kaphi	S R^3 $\underline{G}^1$ (G^3) M^1 P^4 D^3 $\underline{N}^1$ (N^3)	$\underline{N}^1$	--	S G^1 M^1 P^4 D^3
Arshabhi	Re	Sa	Sa Pa	R D N	Bhairavi	S $\underline{R}^2$ (R^4) $\underline{G}^2$ M^2 P^4 $\underline{D}^2$ (D^4) $\underline{N}^2$	$\underline{N}^2$	$\underline{N}^2$ M^2	S P^4 D^2
Gandhari	Ga	Ma	Re Dha	S G M P N	Kalyan	S (R^1) R^4 G^3 M^4 P^4 (D^1) D^4 N^3	N^3	N^3 M^3	D S R^4 G^3 P^4
Madhyama	Ma	Ma	Ga Ni	S R M P D	Khamaj	S R^3 G^3 $\underline{M}^1$ (M^3) P^4 D^3 $\underline{N}^1$ (N^3)	$\underline{N}^1$	$\underline{N}^1$ M^1	P^4 D^3 S R^3 G^3
Panchami	Pa	Ma	Ga Ni	P R	Asavari	S R^4 G^2 (G^4) M^2 P^4 $\underline{D}^2$ (D^4) $\underline{N}^2$	$\underline{D}^2$	$\underline{D}^2$ G^2	S P^4
Dhaivati	Dha	Sa	Pa Sa	R D	' --	S $\underline{R}^2$ (R^4) $\underline{G}^2$ M^1 P^2 (P^4) $\underline{D}^2$ N^2	$\underline{N}^2$	$\underline{N}^2$ G^2	$\underline{M}^1$ S
Naishadi	Ni	Sa	Pa Sa	R G N	Bilaval	S $(\underline{R}^1)$ R^4 G^3 $\underline{M}^1$ (M^3) P^4 D^4 N^3	D^4	D^4 R^4	G^3 $\underline{M}^1$ S



SHRUTI WHEEL



GRAMA WHEEL